THz Links Performance Study for Gamma Turbulence Links with Path Loss and Pointing Errors

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***Abstract*— Over the last years the rapidly growing demands for higher wireless data transfer rates have recently motivated the research community to focus on the exploitation of higher frequency bands, such as the infrared (IR) frequency band and even more recently the terahertz (THz) frequency band (0.3-10 THz) which bridges the gap between millimeter wave (MMW) and IR frequency ranges. Nevertheless, the development of both free space optical (FSO) and THz communication links depends strongly on the randomly varying characteristics of their atmospheric channels along with the stochastic misalignment between transmitter and receiver terminals. Thus, in this work we first introduce Gamma distribution atmospheric turbulence (AT) model in the THz area. In this context, an outage performance comparison between a line of sight (LOS) THz link and a FSO link in terms of outage probability (OP) metric is provided for different AT and stochastic pointing error (PE) conditions. Additionally, the OP for the THz link due to free space path loss (FSPL) and atmospheric attenuation along with stochastic PEs is evaluated. Novel closed-form OP expressions are derived, while proper analytical results reveal and quantify the impact of the above factors. Simulation results are further included to validate our analytical results.**

Keywords –Terahertz; Free Space Optical; Atmospheric Turbulence; Gamma Distribution; Attenuation; Path Loss; Pointing errors; Outage Probability.

I. Introduction

The recent increasing requirement for high speed wireless communication links gave rise to the development of FSO and THz wireless communication systems as very promising alternatives to their radio-frequency (RF) and MMW counterparts. Indeed, both FSO and THz outperform their counterparts in terms of high security level, operation in unregulated frequency ranges, low power consumption, very high information capacities and flexibility for deployment and redeployment. However their availability is degraded due to the effects presented below that mainly arise from the varying nature of the atmospheric medium [1]-[10].

Even in clear weather conditions, random variations of the refractive index of the atmospheric channel that stem from random temperature and pressure inhomogeneities generate AT that causes the so-called scintillation effect which results, in turn, in rapid fluctuations of the intensity of the signal at the receiver's side [4]-[7]. Although THz beams are much less susceptible to scintillation than FSO beams this effect is not a negligible one especially for modern THz links that cover longer propagation distances [4], [8]. However, the THz signal attenuation through humid air is much more severe than FSO. In fact, in comparison with the unavoidable FSPL attenuation, THz waves attenuate more due to molecular absorption especially in the presence of water vapor and in a lesser extent in the presence of oxygen [12]. Moreover, thermal expansion, dynamic wind loads and weak earthquakes result in the sway of high-rise buildings, where THz and FSO terminals are usually placed [13-15]. Thus random intensity variations of the received signal due to PEs emerge.

While the influence of PEs and AT has been extensively studied for the FSO links in the open literature [13-19], little work has been done in the THz area. The impact of THz beams misalignment has been discussed in [20-22]. However, deterministic models had been assumed which cannot accommodate the stochastic nature of building sway. Based on the PEs model which was first proposed for the FSO in [13], the stochastic nature of PEs for THz links has been recently first incorporated in [2] and then in [8], [23-25]. Similarly, based on several well-known AT distribution models for the FSO links, the probabilistic turbulence-induced fading has been recently first modeled for THz links in [8]. Motivated by these facts in this work we introduce in the THz region the Gamma distribution weak turbulence-induced fading model as a realistic alternative to the more complex lognormal (LN) distribution model which has been utilized recently in [8]. Note that the more compact Gamma distribution model has been already proved to accurately emulate weak AT conditions for the FSO links [16-18]. Thus, the OP for a typical THz link due the combined Gamma modeled turbulence-induced and stochastic PEs is investigated in comparison with the corresponding OP for the FSO configuration. Moreover, the OP due to the total attenuation and stochastic PEs is also evaluated for the THz link.

II. System And Channel Model

A. Signal Model

It is assumed that both LOS THz and FSO systems under investigation employ On-Off Keying (OOK) modulation format which is commonly utilized in the commercial field mainly due to its simplicity. After traversing the atmospheric channel, the received signal can be expressed as

$$y = hx + n\,, \tag{1}$$

where h denotes the total channel state, $x = \{0 \text{ or } 2P\}$ is the information signal with P being the average transmitted signal power and n represents the additive white Gaussian noise (AGWN) with variance, σ_n^2 [8].

The total channel coefficient can be written as

$$h = h_l h_p h_a\,, \tag{2}$$

where h_l, h_p and h_a represent the dependence of the signal intensity value due to the deterministic total attenuation, PEs and AT, respectively [18].

B. Total Attenuation

The deterministic attenuation term for the THz link configuration can be expressed as

$$h_l = h_{fl} h_{wl}\,, \tag{3}$$

with h_{fl} and h_{wl} standing for the attenuation due to the free space path loss and the attenuation due to the water vapor, respectively. The former factor is expressed according to Friis equation as

$$h_{fl} = \frac{c\sqrt{G_t G_r}}{4\pi f z}\,, \tag{4}$$

where c is the speed of the light in the free space, f is the operating frequency of the carrier wave, z is the link length and G_t, G_r are standing for the transmission and reception antenna gains, respectively [25]. Additionally, the latter attenuation factor can be expressed as, [13]:

$$h_{wl} = \exp\left(-a_w z\right), \tag{5}$$

where a_w is the attenuation coefficient in m^{-1} while z is expressed in meters. Specifically, since water vapor attenuation dominates a_w practically represents the attenuation coefficient due to water vapor which at 20°C surface temperature and for $f \le 350$ GHz is determined as [26]

$$a_w\left(\mathrm{dB/km}\right) = f^2\rho\left[0.067 + \frac{2.4}{\left(f-22.3\right)^2+6.6} + \frac{7.33}{\left(f-183.5\right)^2+5} + \frac{4.4}{\left(f-323.8\right)^2+10}\right]10^{-4}\,, \tag{6}$$

where f is now expressed in GHz and ρ denotes the water vapor concentration in $\mathrm{g/m^3}$.

C. Atmospheric Turbulence Model

The probability density function (PDF) for random variable h_a is obtained through Gamma distribution as [17]

$$f_{h_a}\left(h_a\right) = h_a^{\zeta-1}\zeta^{\zeta}\exp\left(-\zeta h_a\right)\Gamma^{-1}(\zeta)\,, \tag{7}$$

where $\Gamma(.)$ is the gamma function [27, (06.05.02.0001.01)] and ζ represents the parameter of the Gamma distribution which is given as [18]

$$\zeta = \left[\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\alpha\beta}\right]^{-1} \tag{8}$$

The parameters α and β are expressed as [16]:

$$\alpha = \left[\exp\left(\frac{0.49\delta^2}{\left(1+0.18d^2+0.56\delta^{12/5}\right)^{7/6}}\right)-1\right]^{-1}$$

$$\beta = \left[\exp\left(\frac{0.51\delta^2\left(1+0.69\delta^{12/5}\right)^{-5/6}}{\left(1+0.9d^2+0.62d^2\delta^{12/5}\right)^{5/6}}\right)-1\right]^{-1}, \tag{9}$$

where $d = 0.5D\sqrt{2\pi\lambda^{-1}z^{-1}}$, $\delta^2 = 0.5C_n^2\kappa^{7/6}z^{11/6}$, $\kappa=2\pi/\lambda$, D stands for the receiver's aperture diameter, λ, is the operational wavelength, and C_n^2 is the refractive index structure parameter which varies between 10^{-17} $\mathrm{m}^{-2/3}$ and 10^{-13} $\mathrm{m}^{-2/3}$ for weak to strong AT conditions, respectively [17].

D. Pointing Errors Model

The PDF for random variable h_p is obtained as [2], [13]

$$f_{h_p}\left(h_p\right) = \psi^2 A_0^{-\psi^2} h_p^{\psi^2-1},\quad 0 \le h_p \le A_0, \tag{10}$$

where $\psi = w_{eq}/2\sigma$, with σ and w_{eq} standing for the PE displacement standard deviation, i.e. the spatial jitter, and the equivalent beam width, respectively. Additionally, $w_{eq} = \left[\sqrt{\pi}\mathrm{erf}(v)w^2/2v\exp(-v^2)\right]^{1/2}$, while $A_0 = \mathrm{erf}^2(v)$ is the fraction of the collected power at $r = 0$ with r being the radius of receiver aperture, $v = \sqrt{\pi}r/\sqrt{2}\,w$, and w denotes the beam waist on the receiver plane at propagating distance z [8].

E. Joint Impact of Turbulence and Pointing Errors

Following the procedure which is presented in [18] we get

$$f_h\left(h\right) = \frac{\psi^2\zeta}{A_0\Gamma\left(\zeta\right)} G_{1,2}^{2,0}\left(\frac{\zeta h}{A_0}\middle|\begin{matrix}\psi^2\\ \psi^2-1,\ \zeta-1\end{matrix}\right), \tag{11}$$

where $G[.]$ denotes the Meijer function [27, (07.34.02.0001.01)].

By integrating (11) and after using [27, (07.34.21.0084.01)] the corresponding cumulative density function (CDF) is obtained as [16], [18]

$$F_h\left(h\right)=\int_0^h f_u\left(u\right)\,du=\frac{\psi^2}{\Gamma\left(\zeta\right)}G_{2,3}^{2,1}\left(\frac{\zeta h}{A_0}\middle|\begin{array}{ccc} & 1, & \psi^2+1 \\ \psi^2, & \zeta, & 0\end{array}\right). \quad (12)$$

F. Joint Impact of Attenuation and Pointing Errors

There is a strong inverse correlation between AT strength and attenuation [13]. Thus, in order to focus on the joint impact of attenuation and pointing errors we set $h_a = 1$. Considering (2)-(6) and after a random variable transformation the joint PDF of the random variable h is now obtained as:

$$f_h\left(h\right)=\psi^2 A_0^{-\psi^2} h_l^{-\psi^2} h^{\psi^2-1},\quad 0\le h\le A_0 h_l. \quad (13)$$

Similarly, by integrating (14) and considering also (3) the corresponding CDF is obtained as

$$F_h\left(h\right)=A_0^{-\psi^2} h_{fl}^{-\psi^2} h_{wl}^{-\psi^2} h^{\psi^2}. \quad (14)$$

III. Outage Probability Estimation

A. Outage Probability due to turbulence and pointing errors

The OP denotes the probability that the instantaneous electrical SNR, γ, falls below a specific value, γ_{th}, which represents the receiver's sensitivity threshold. Additionally, $\gamma = 2P^2h^2/\sigma_n^2$ while the average electrical SNR, is $\mu = 2P^2(E[h])^2/\sigma_n^2$, with $E[h] = A_0(1+\psi^{-2})^{-1}$ being the expected value of h [16]. Therefore, the OP is expressed as

$$P_{out} = \Pr\left(h<h_{th}\right)=F_h\left(h_{th}\right). \quad (15)$$

By using (12), (15) along with the above SNR expressions we obtain the OP due to turbulence and PEs as

$$P_{out.T}=\frac{\psi^2}{\Gamma\left(\zeta\right)}G_{2,3}^{2,1}\left(\sqrt{\frac{\gamma_{th}}{\mu}}\frac{\psi^2}{\psi^2+1}\middle|\begin{array}{ccc} & 1, & \psi^2+1 \\ \psi^2, & \zeta, & 0\end{array}\right), \quad (16)$$

where γ_{th}/μ is the normalized average electrical SNR.

B. Outage Probability due to atteniuation and pointing errors

By using (14), (15) along with the above SNR expressions we obtain the OP due to attenuation and PEs as

$$P_{out.A}=A_0^{-\psi^2} h_{fl}^{-\psi^2} h_{wl}^{-\psi^2}\left(\frac{\gamma_{th}N_0}{P}\right)^{\frac{\psi^2}{2}}, \quad (17)$$

where $N_0 = \sigma_n^2/2$ and h_{fl}, h_{wl} are obtained through (4)-(6).

IV. Analytical Results

It is assumed that the operational wavelength for the FSO is 1.55μm while the operational frequency for the THz is 0.3THz. The aperture receiver radius, r, is equal to 0.05m for the FSO detector while for the THz antennas it is 0.15m or 0.7m for $G_t = G_r = G = 55$dBi or 70dBi, respectively [8]. Additionally $z = 100$m, $\sigma_n = 10^{-7}$A/Hz, while $C_n^2 = 5\times10^{-16}$ m$^{-2/3}$ or 9×10^{-16} m$^{-2/3}$, and $w/r = 19$ with $(\sigma/r, \psi) = (14, 0.68)$ or $(\sigma/r, \psi) = (12, 0.79)$ for strong and weak PEs, respectively. It is also assumed 20°C surface temperature with $\rho = 7.5$ g/m^3 or 9.5 g/m^3 and $\gamma_{th} = 1$ or 5 using (16) and (17) respectively [2]. Under these circumstances, proper analytical results are illustrated below.

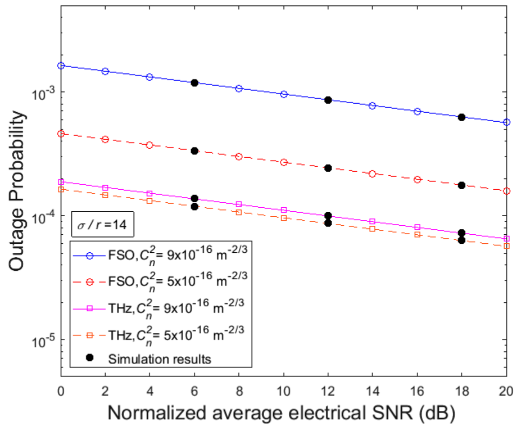

Fig. 1. *OP for FSO and THz due to turbulence and strong PEs.*

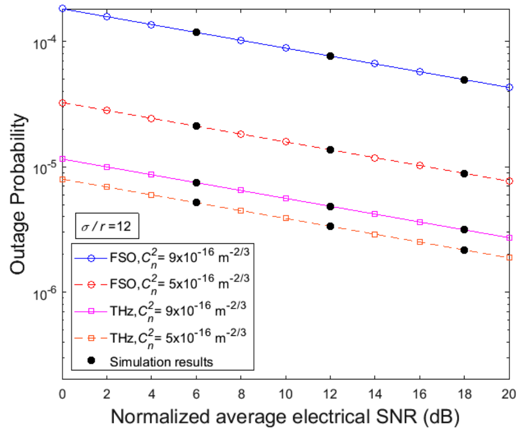

Fig. 2. *OP for FSO and THz due to turbulence and weak PEs.*

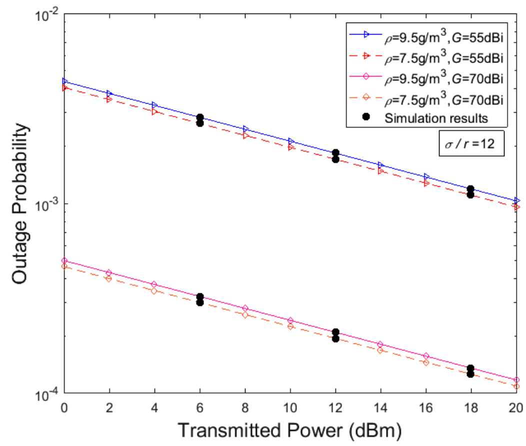

Fig. 3. *OP for THz due to attenuation and PEs.*

Figs 1 and 2 highlight that FSO is more vulnerable than THz to atmospheric turbulence variations especially for stronger turbulence, stronger PEs and lower SNR values. Fig. 3 depicts however that attenuation due to FSPL and water vapor drastically affects the OP for THz especially for larger water vapor concentrations along with lower transmitted power values and lower antenna gain values. Note that the qualitatively behaviors revealed above are consistent with findings of [2], [8], [16].

V. Conclusions

In this work an OP performance analysis has been presented including the prime factors that affect FSO and THz outage performance. The proposed Gamma distribution for THz weak AT modeling has been proved to be valid and accurate enough. Additionally, the feasibility of establishing realistic THz links has been validated, especially for low air humidity conditions. Towards this direction particular attention should be given to the design of hybrid THz/ FSO systems of greater total availability, performance and coverage area in the near future.

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References

[1] I.F. Akyildiz, J.M. Jorne, and C. Han, “Terahertz band: Next frontier for wireless communications. *Physical communication*, vol. 12, pp. 16-32, 2014.

[2] A.A.A Boulogeorgos, E.N. Papasotiriou, and A. Alexiou, “Analytical performance assessment of THz wireless systems,” *IEEE Access*, vol. 7, pp. 11436-11453, 2019.

[3] T. Kürner, and S. Priebe, “Towards THz communications-status in research, standardization and regulation,” *Journal of Infrared, Millimeter, and Terahertz Waves* ,vol. 35, no. 1, pp. 53-62, 2014.

[4] J. Federici, and L. Moeller, “Review of terahertz and subterahertz wireless communications,” *Journal of Applied Physics*, vol. 107, no. 11, p. 6, 2010.

[5] G.K. Varotsos, H.E. Nistazakis, A.N. Stassinakis, C.K. Volos, V. Christofilakis, and G.S. Tombras, “Mixed Topology of DF Relayed Terrestrial Optical Wireless Links with Generalized Pointing Errors over Turbulence Channels,” *Technologies*, vol. 6, no. 4, p. 121, 2018.

[6] G.K. Varotsos, H.E. Nistazakis, and G.S. Tombras, “OFDM RoFSO Links with Relays Over Turbulence Channels and Nonzero Boresight Pointing Errors,” *J. Commun*., vol. 12, no. 12, p. 644, 2017.

[7] N.D. Chatzidiamantis, H.G. Sandalidis, G.K. Karagiannidis, and M. Matthaiou, “Inverse Gaussian modeling of turbulence-induced fading in free-space optical systems,” *Journal of Lightwave Technology*, vol. 29, no. 10, pp. 1590-1596, 2011.

[8] M. Taherkhani, Z.G. Kashani, and R.A. Sadeghzadeh, “On the performance of THz wireless LOS links through random turbulence channels,” *Nano Communication Networks*, vol. 23, p. 100282, 2020.

[9] K. Su, L. Moeller, R.B. Barat, and J.F. Federici, “Experimental comparison of terahertz and infrared data signal attenuation in dust clouds,” *JOSA A*, vol. 29, no. 11, pp. 2360-2366, 2012.

[10] J. Ma, F. Vorrius, L. Lamb, L. Moeller, and J.F. Federici, “Experimental comparison of terahertz and infrared signaling in laboratory-controlled rain,” *Journal of Infrared, Millimeter, and Terahertz Waves*, vol. 36, no. 9, pp. 856-865, 2015.

[11] J.F. Federici, J. Ma, and L. Moeller, “Weather impact on outdoor terahertz wireless links,” Proc. of the Second Annual International Conference on Nanoscale Computing and Communication, pp. 1-6, September 2015.

[12] Y. Yang, M. Mandehgar, and D. Grischkowsky, “Determination of the water vapor continuum absorption by THz-TDS and molecular response theory,” *Optics Express*, vol. 22, no. 4, pp. 4388-4403,2014.

[13] A. Farid, and S. Hranilovic, “Outage capacity optimization for free-space optical links with pointing errors”, *J. Lightw. Techn.*, vol. 25, no. 7, pp. 1702-1710, 2007.

[14] T.A. Tsiftsis, H.G. Sandalidis, G.K. Karagiannidis and M. Uysal, “Optical wireless links with spatial diversity over strong atmospheric turbulence channels”, *IEEE Trans. on Wireless Comm.*, vol. 8, no. 2, pp. 951-957, 2009.

[15] W. Gappmair, S. Hranilovic, and E. Leitgeb, “Performance of PPM on terrestrial FSO links with turbulence and pointing errors”, *IEEE Communications Letters*, vol. 14, no. 5, pp. 468-470,2010.

[16] G.K. Varotsos, H.E. Nistazakis, A.N. Stassinakis, G.S. Tombras, V. Christofilakis, and C.K.Volos, “ Outage performance of mixed, parallel and serial DF relayed FSO links over weak turbulence channels with nonzero boresight pointing errors,” in *7th International Conference on Modern Circuits and Systems Technologies (MOCAST)* IEEE, pp. 1-4, May 2018.

[17] A.N. Stassinakis, H.E. Nistazakis, G.K. Varotsos, G.S. Tombras, A.D. Tsigopoulos, and V. Christofilakis, “Outage capacity estimation of FSO links with pointing errors over gamma turbulence channels,” in *5th International Conference on Modern Circuits and Systems Technologies (MOCAST)* IEEE, pp. 1-4, May 2016.

[18] G.K. Varotsos, H.E. Nistazakis, C.K. Volos, and G.S. Tombras, “FSO links with diversity pointing errors and temporal broadening of the pulses over weak to strong atmospheric turbulence channels,” *Optik*, vol.127, no. 6, pp. 3402-3409, 2016.

[19] G.D. Roumelas, H.E. Nistazakis, K. Aidinis, G.K. Varotsos, and E. Leitgeb, “Total Outage Probablity of a Multihop Hybrid FSO/MMW System,” in *IEEE International Symposium on Signal Processing and Information Technology (ISSPIT)*, pp. 1-5, December 2019.

[20] S. Priebe, M. Jacob, and T. Kürner, “The impact of antenna directivities on THz indoor channel characteristics,” in *6th European Conference on Antennas and Propagation (EUCAP)* IEEE, pp. 478-482, March 2012.

[21] A. Ekti et al., “Statistical modeling of propagation channels for terahertz band,” in *IEEE Conference on Standards for Communications and Networking (CSCN)*, pp. 275-280, September 2017.

[22] C. Han, and I.F. Akyildiz, “Three-Dimensional End-to-End Modeling and Analysis for Graphene-Enabled Terahertz Band Communications,” *IEEE Transactions on Vehicular Technology*, vol. 66, no. 7, pp. 5626-5634, 2017.

[23] A.A.A Boulogeorgos, and A. Alexiou, “Outage probability analysis of THz relaying systems,” in *IEEE 31st Annual International Symposium on Personal, Indoor and Mobile Radio Communications*, pp. 1-7, August 2020.

[24] J. Kokkoniemi, A.A.A. Boulogeorgos, M. Aminu, J. Lehtomäki, A. Alexiou, and M. Juntti, “Impact of beam misalignment on THz wireless systems,” *Nano Communication Networks*, vol. 24, pp. 100302, 2020.

[25] E.N. Papasotiriou, A.A.A Boulogeorgos, and A. Alexiou, “Performance analysis of THz wireless systems in the presence of antenna misalignment and phase noise,” *IEEE Communications Letters*, vol. 24, no. 6, pp. 1211-1215, 2020.

[26] L.J Ippolito, Radiowave propagation in satellite communications. New York, 1986.

[27] The Wolfarm Functions Site, 2008. [Online] Available: http://functions.wolfarm.com.