Study of a Chaotic Circuit with a Physical Memristor as a Nonlinear Resistor

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***Abstract*—In this paper, a novel approach of the KNOWM's physical memristor behavior is introduced. It has been experimentally observed that this memristor, under certain conditions, can behave as a static nonlinear resistor. This inherent property of the KNOWM's memristor prompts its use as a nonlinear reistor in chaotic circuits. Therefore, in this work, for the first time, the KNOWM memristor is used as a static nonlinear resistor in a well-known chaotic oscillator circuit. In order to examine circuit's dynamical behavior a host of nonlinear simulation tools, such as phase portraits, bifurcation and continuation diagrams, as well as maximal Lyapunov exponent diagram, are used. Interesting phenomena related to chaos theory are observed. More specifically, period-doubling route to chaos, crisis phenomena, antimonotonicity, hysteresis and coexisting attractors, are investigated.**

***Index Terms*—Antimonotonicity, bifurcations, chaos, coexisting attractors, hysteresis, KNOWM memristor, nonlinear resistor**

I. Introduction

Chua in a seminal paper in 1971 [1] identified a theoretical symmetry between the linear resistor (voltage versus current), linear capacitor (voltage versus charge), and linear inductor (magnetic flux linkage versus current). From this symmetry he inferred the characteristics of a fourth fundamental nonlinear circuit element, linking magnetic flux and charge, which he called memristor.

Therefore, the memristor, is a nonlinear two-terminal electrical component relating electric charge and magnetic flux linkage [2]. Moreover, in contrast to a linear (or nonlinear) resistor the memristor has a dynamic relationship between current and voltage including a memory of past voltages or currents. In more details, in a memristor, when the current flows in one direction, its resistance decreases and vice versa [3]. When the current flow stops, memristor retains its final state. As a consequence the $i - v$ characteristic curve of a memristor has a form of a hysteresis loop, which is pinched in the origin. This is one of the well-known fingerprints of the memristive elements driven by bipolar periodic signals of any amplitude and frequency [4].

However, the *ideal* memristor that Chua introduced was mainly of theoretic interest. On the other hand the *generic* or *extended* memristors, that introduced by Chua and Kang in 1976 [5] were used to describe physical devices as memristors.

Therefore, until the end of the first decade of 21st century, memristor had received little attention. However, in 2008, a team in Hewlett-Packard labs built the first electronic passive memristor [6]. Furthermore, in the last five years memristors from the KNOWM Inc. are commercially available. The KNOWM memristor [7] material stack is based on a mobile metal ion conduction through a chalcogenide material that has undergone a metal-catalyzed chemical reaction that creates channels which constrain the flow of metal ions.

The aforementioned intrinsic nonlinear characteristic of memristor has given to the research community the idea that it could be exploited in implementing novel chaotic circuits and systems with complex dynamics. In this direction the last three years a few implementations of chaotic circuits with physical memristors have been proposed [8]–[10].

In this work, a different approach regarding the use of the KNOWM physical memristor has been followed. It has been observed that the specific memristor for low frequencies behaves approximately as a static nonlinear resistor. This drawback of the KNOWM memristor could be a real interesting feature, due to the fact that it could be used as a nonlinear resistor in chaotic circuits. Therefore, by using the experimental data of the KNOWM memristor's nonlinear $i - v$ characteristic curve, the mathematical description of the nonlinear resistor that this memristor can be used is calculated. Next, the memristor as the proposed nonlinear resistor, is used in the well-known Shinriki chaotic oscillator circuit. Finally, the numerical investigation of the circuit's dynamics is presented. This investigation is based on the simulation results, which are produced by using numerical tools, such as phase portraits, maximal Lyapunov exponents [11]–[13], bifurcation diagrams [14], and continuation diagrams [15].

The paper is organized as follows. In Section II, the KNOWM memristor's characteristic curve for low frequencies as well as the proposed chaotic circuit are introduced. In Section III, the numerical investigation of the circuit's dynamics is presented. Finally, the conclusions and some thoughts for future works are discussed in Section IV.

II. The Proposed Chaotic Circuit with the Physical Memristor

The $i-v$ characteristic curve of one of the eight KNOWM memristors, which are contained in the 16-pin ceramic DIP package, is depicted in Fig. 1. This $i-v$ curve (with yellow color) has been captured by using the Analog Discovery 2 USB oscilloscope by using a sinusoidal signal of amplitude 2.4 V and frequency 10 Hz. From Fig. 1 is obtained that the pinched hysteresis loop of the memristor's $i-v$ characteristic curve has been shrinks so much that it could be considered as a simple nonlinear curve. Moreover, the curve (with blue color) is the $i-v$ characteristic of the memristor in series with a resistor.

Furthermore, the experimental data of the memristor's characteristic $i_M - v_M$ curve of Fig. 1, which are also captured in a PC, are used in order to calculate the mathematical formula that fits better to it. Therefore, by using the least squares method the following equation is produced,

$$i_M = 0.0125 \cdot e^{-0.00744 v_M} \cdot \sinh(0.68 \cdot v_M) \qquad (1)$$

with $R^2 = 0.9996$, which is presented in Fig. 2. So, for low frequencies the KNOWM memristor could be used as a nonlinear resistor, which its $i_M - v_M$ characteristic curve is described by Eq. 1.

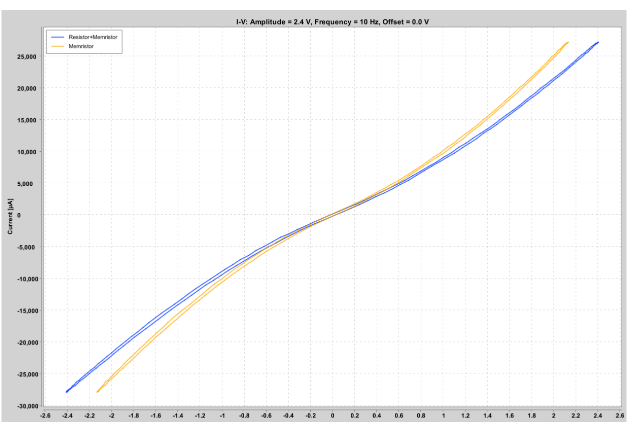

Fig. 1: Experimental $i_M - v_M$ characteristic curve of the memristor.

This nonlinear resistor is used in the well-known Shinriki circuit [16], by replacing the nonlinear positive conductance with the KNOWM memristor. The circuit consists of one negative conductance and a resonant circuit, with resonant frequency,

$$f_0 = \frac{1}{2\pi\sqrt{LC_2}} \qquad (2)$$

Therefore, the schematic of the proposed circuit with the physical memristor as the proposed nonlinear resistor, is depicted in Fig. 3. This circuit is described by the following set of dimensionless equations:

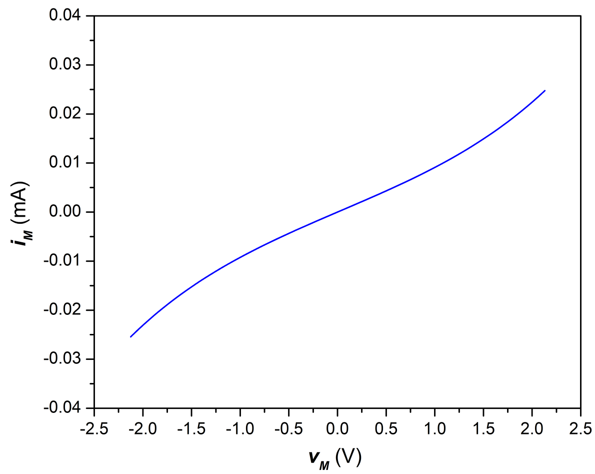

Fig. 2: Fitting curve with Eq. 1 of the experimental $i_M - v_M$ characteristic.

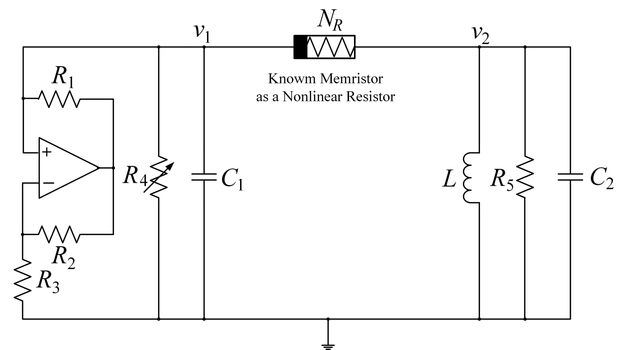

Fig. 3: The proposed chaotic circuit.

$$\begin{aligned} \dot{x} &= \eta[(\alpha-\beta)x + i] \\ \dot{y} &= -z - \gamma y - i \\ \dot{z} &= y \end{aligned} \qquad (3)$$

where, i represents the $i_M - v_M$ nonlinear characteristic curve of Eq.(1). Also, the circuit's normalizing variables and parameters are:

$$x = \frac{v_1}{v_{ref}}, y = \frac{v_2}{v_{ref}}, z = \frac{\rho i_L}{v_{ref}}, i = \frac{\rho i_M}{v_{ref}}, \tau = \frac{t}{\sqrt{LC_2}}$$
$$\rho = \sqrt{\frac{L}{C_2}}, \eta = \frac{C_2}{C_1}, \alpha = \frac{\rho}{R_3}, \beta = \frac{\rho}{R_4}, \gamma = \frac{\rho}{R_5} \qquad (4)$$

Next, the circuit of Fig. 3, is numerically studied by using the following values of circuit's elements: $L = 0.5H$, $C_1 = 50.66\mu F$, $C_2 = 506.66\mu F$, $R_1 = R_2 = 5.6k\Omega$, $R_3 = 0.109k\Omega$, $R_5 = 0.1k\Omega$ and R_4 - tunable, while the power supply is ± 10 V. With this set of elements' values, the system's (3) dimensionless parameters values are fixed to:

$\alpha = 0.288$, $\gamma = 0.314$ and $\eta = 10$, while β is the control parameter.

III. Numerical Results

In this section, the dynamical behavior of the proposed system (3), for different values of the control parameter β, is investigated. Generally, the system has rich dynamics that include periodic and chaotic behavior as well as other interesting phenomena related to chaos theory.

Figure 4 presents the bifurcation diagram of variable x versus the value of parameter β, with initial conditions $x_0 = 0.05$, $y_0 = 0.01$ and $z_0 = 0$.

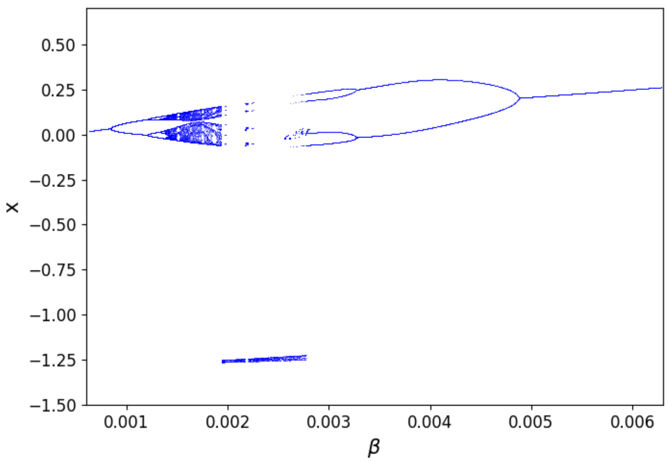

Fig. 4: Bifurcation diagram of x versus the dimensionless parameter β.

From this diagram the rich dynamical behavior of system (3) is obtained. There are regions where the system oscillates periodically and regions where the system oscillates chaotically. This behavior is also confirmed from the maximal Lyapunov exponent (LE_{max}), which is presented in Fig. 5. It is clear that when the Lyapunov exponent is positive the existence of chaotic behavior is observed, while when the Lyapunov exponent is not positive the system has a periodic behavior. In more details, as the value of parameter β increases a period doubling route is observed and the system goes from a period-1 behavior to a chaotic behavior. Also, for $\beta = 0.0019383$ a sudden jump from the upper part of the diagram to the lower part is observed. This phenomenon is known as hysteresis.

Moreover, from Fig. 4 the antimonotonicity phenomenon [17]–[20] can be revealed. According to this phenomenon, the system enters to chaos with the well-known period doubling route and exits from the chaos following the reverse period doubling route. Furthermore, in Fig. 6(a) the respective continuation diagram is presented. The continuation diagram is produced by using as initial conditions in each step the final states of the previous step. Also, in Fig. 6(b) the zoom in the lower part of the continuation diagram, reveals a period doubling route to chaos from a period-1 state. The conclusion of the comparison of the bifurcation with the continuation diagram, is that the system presents the phenomenon of coexisting attractors. Two coexisting attractors (periodic and chaotic) for $\beta = 0.001570$ and for different initial conditions are presented in Fig. 7 respectively.

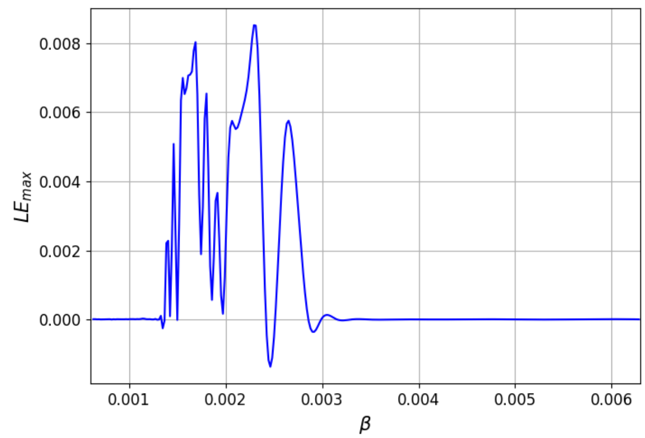

Fig. 5: Maximal Lyapunov exponent diagram versus the dimensionless parameter β.

IV. Conclusion

In this work a three dimensional autonomous circuit based on a physical memristor was simulated. The novelty of this work was the use of the physical KNOWM memristor as a simple nonlinear resistor. For this reason, a mathematical function that describes the $i-v$ characteristic of the memristor was produced through the fitting of its experimental data.

The autonomous circuit presented rich dynamical behavior. Chaotic and periodic behavior were observed. Moreover, the system presented phenomena concerning chaos theory, such as route to chaos through the mechanism of period doubling, the hysteresis and antimonotonicity phenomena, as well as coexisting attractors. Furthermore, this approach presents the usefulness of the memristor as a nonlinear resistor in electrical circuits.

Finally as a further study of this work will be examined the implementation of the circuit and the experimental confirmation of the numerical results.

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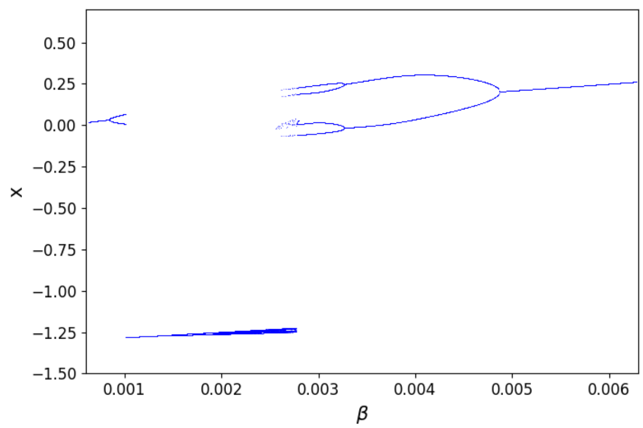
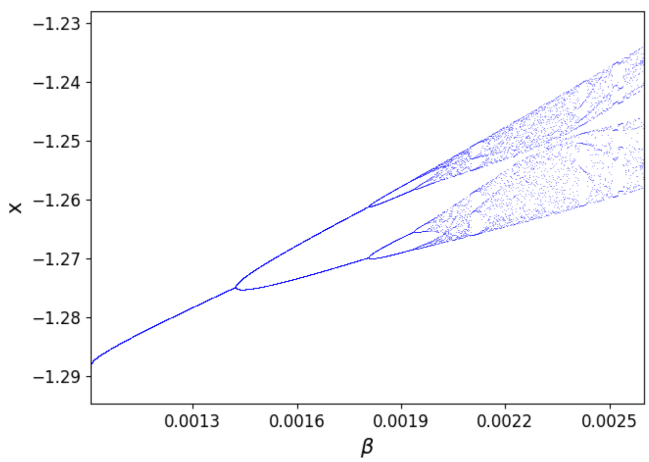
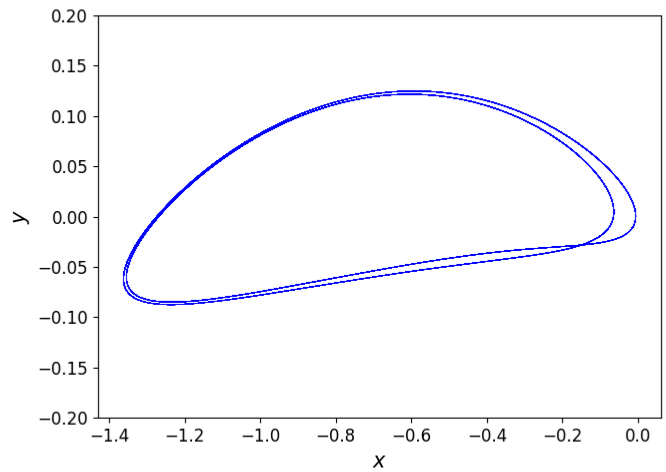


(a)

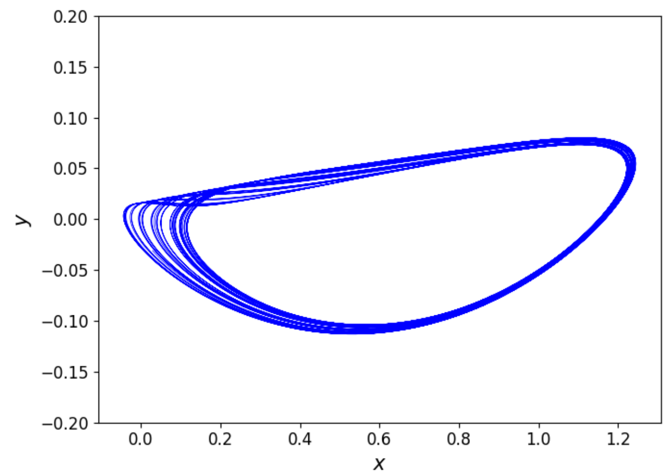


(b)

Fig. 6: (a): Continuation diagram of *x* versus the dimensionless parameter β and (b) zoom in the lower region of the continuation diagram.



(a)



(b)

Fig. 7: Phase space in *x*-*y* plane, for $\beta = 0.00157$ and for (a): $x_0 = -0.5751$, $y_0 = 0.1213$, $z_0 = -0.2258$ and (b) $x_0 = 0.05$, $y_0 = 0.01$, $z_0 = 0.0$.

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