A Study on the Influence of Pseudo–Acceleration Vector for Accelerometer Calibration

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***Abstract*—In order to use the accelerometer with high accuracy, it is generally necessary to calibrate the accelerometer and correct the observation data. The authors proposed a method for easily calibrating the accelerometer by generating the pseudo–acceleration vector from a limited observed acceleration vector. However, the analysis of how the pseudo–acceleration vector works on the calibration algorithm was insufficient, and the results are reported in this paper.**

***Index Terms*—Accelerometer, Calibration, Pseudo data, Non–linear optimization**

I. Introduction

One of the sensors used to quantify motion is the accelerometer. In recent years, with the development of MEMS (Micro Electro Mechanical Systems) technology, multi-axis accelerometers, which have multiple axes in one sensor package, have become popular and are widely installed in devices related to motion, such as smartphones, drones, automobiles, game machines and robots.

The measured values obtained from multi–axis accelerometers contain errors, and typical examples of these errors are (1) Non–orthogonality between axes that occurs during sensor manufacturing, called misalignment, (2) The ratio of the output (measured value of the sensor) to the input (actual acceleration value), called the scale factor, (3) Offset error in which the output appears even though the input acceleration is zero, called zero–g bias and (4) Steady white Gaussian noise due to thermal noise, dark current, etc. Since the characteristics of these error factors differ for each sensor, it is essential to correct them by calibration in order to use the sensor accurately.

There are two types of calibration methods for multi–axis accelerometers: using external devices and using only the sensor itself. The method combining the Kalman filter and the Six–Position method [1] is to install an acceleration sensor on a turntable, which is a rotating device that can be finely adjusted in angle, and measure the acceleration data for a total of six positions while controlling each axis (x, y, z) of the sensor so that they are precisely oriented vertically upward and downward. This method is used to determine the misalignment, scale factor, and zero-g bias. This method is suitable for industrial accelerometers where accuracy is required mainly because of its high accuracy in estimating error factors. Xiao et al. proposed a more general and simple calibration method [2] that uses information obtained from a camera-based visual inertial system to calibrate accelerometers. This method can calibrate accelerometers automatically and in real time using a monocular camera. Wang et al. have proposed a method to improve the estimation accuracy of error factors by using a Kalman filter on acceleration data measured with a low-precision, inexpensive turntable [3]. These methods require external devices other than the accelerometer to be prepared.

On the other hand, a method has been proposed to determine error factors by optimization using data measured by tilting the sensor to any of 36 to 50 patterns without requiring external devices such as turntables other than accelerometers [4]. Although it enables accurate estimation of correction parameters without the use of external equipment, there are some issues such as the large number of postures required and the time required for data acquisition. To solve this problem, Ariyama et al. proposed a simple method to calibrate the sensor by measuring only the acceleration data of six postures in which each axis is vertically upward or downward and pseudo–generating the acceleration data of the unmeasured postures [5].

In this paper, we report the results of a study of the method in [5], because it was not sufficiently analyzed how pseudo–acceleration vector acted on the calibration algorithm.

II. Calibratoin method using pseudo–acceleration vector

In this section, we explain a calibration method for the 3–axis accelerometer [5].

A. Error factor and correction model of accelerometer

Figure 1 shows a coordinate system of the 3–axis accelerometer assumed in this study. The 3–axis accelerometer has an accelerometer element for each axis, and each element can measure acceleration in the axial direction. The coordinate system composed by the accelerometer element is called the accelerometer frame, and it axes define as X^S, Y^S, and Z^S, respectively. In addition, the coordinate system of the 3–axis accelerometer housing is called the body frame, and it axes define as X^B, Y^B, and Z^B, respectively. Furthermore, each axes of the body frame is assumed to be orthogonal to each other.

There are various factors that can cause errors in the 3–axis accelerometer, in this study, we will consider three factors: *misalignment*, *zero–g bias*, and *scale factor*. The misalignment

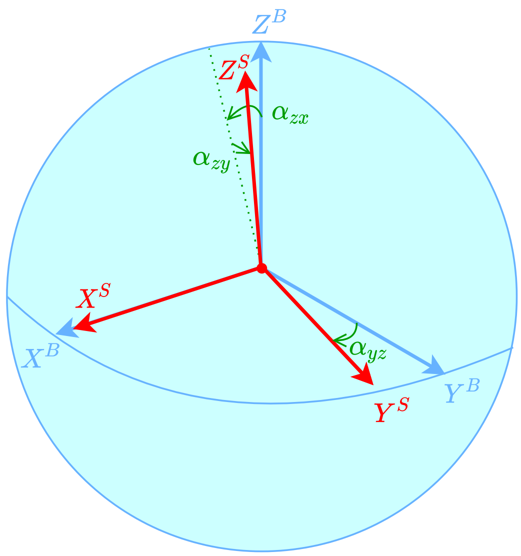

Fig. 1. Accelerometer coordinate system.

refers to the non–orthogonality of the accelerometer frame. Ideally, the axes of the accelerometer frame should be orthogonal to each other, however, they are non–orthogonal in reality caused by the placement error of the accelerometer element in the manufacturing process. In this study, the misalignment is defined as follows:

1) X^S and X^B coinside,
2) Y^S lies in the plan spanned by X^B and Y^B.

The zero–g bias is also the placement error caused by the manufacturing process, and is an error that acceleration is detected even when no acceleration is applied to the sensor. The scale factor is the ratio of the input acceleration to the accelerometer output. The ratio of input to output is describe "1" ideally, however, corrections are necessary due to error effects.

Based on the above, when the error–corrected 3–axis acceleration vector is $\boldsymbol{a}^O = [a_x^O, a_y^O, a_z^O]^T$, the error correction model can be expressed as

$$\boldsymbol{a}^O = \boldsymbol{T}^a \boldsymbol{K}^a (\boldsymbol{a}^S + \boldsymbol{b}^a), \tag{1}$$

where $\boldsymbol{a}^S = [a_x^S, a_y^S, a_z^S]^T$ is the acceleration vector observed by the accelerometer. Also, $\boldsymbol{T}^a$ is the misalignment matrix, and considering the non–orthogonality of the axes described above, we have

$$\boldsymbol{T}^a = \begin{bmatrix} 1 & -\alpha_{yz} & \alpha_{zy} \\ 0 & 1 & -\alpha_{zx} \\ 0 & 0 & 1 \end{bmatrix}. \tag{2}$$

Note that the element α_{ij} means the rotation of the i–axis around the j–axis. Furthermore, $\boldsymbol{K}^a$ and $\boldsymbol{b}^a$ represent the scale factor matrix and the zero–g bias vector as follow:

$$\boldsymbol{K}^a = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & s_z \end{bmatrix}, \tag{3}$$

$$\boldsymbol{b}^a = [b_x \ b_y \ b_z]^T. \tag{4}$$

The purpose of this calibration method is to estimate the optimal correction parameters $\boldsymbol{u}^{acc} = [\alpha_{yz}, \alpha_{zy}, \alpha_{zx}, s_x, s_y, s_z, b_x, b_y, b_z]^T$.

B. Estimation of correction parameters

In order to estimate the correction parameters, we define the following function using (1):

$$\boldsymbol{a}^O = h(\boldsymbol{a}_k^S, \boldsymbol{u}^{acc}) = \boldsymbol{T}^a \boldsymbol{K}^a (\boldsymbol{a}^S + \boldsymbol{b}^a), \tag{5}$$

and we use the following evaluation function:

$$J(\boldsymbol{u}^{acc}) = \sum_{k=1}^{N} \left(||\boldsymbol{g}||^2 - ||h(\boldsymbol{a}_k^S, \boldsymbol{u}^{acc})||^2 \right)^2, \tag{6}$$

where $||\boldsymbol{g}||^2$ is the actual magunitude of the local gravity vector. Using the accelerations obtained from the various postures, the parameter that minimizes (6) is the optimal correction parameter. The procedure for estimating correction parameters using the evaluation function is as follows:

1) To calculate the detection parameter of stationary section, the sensor is placed in a stationary state for a certain period of time.
2) Fix the accelerometer at an arbitrary posture and maintain the posture for a certain period of time. This operation is repeated N times to obtain the acceleration at various postures.
3) From the time series of acceleration data obtained in 2), the stationary section of each posture pattern is cut out and averaged over the section to remove noise. The averaged acceleration is $\boldsymbol{a}_k^S (k = 1 \sim N)$ and is used in the evaluation equation (6).
4) Estimate the optimal correction parameter $\boldsymbol{u}_{opt}^{acc}$ using the Levenberg–Marquardt method (LM method) for the evaluation function (6).

C. Generating pseudo–acceleration vector

This calibration method generates a pseudo–acceleration vector using 6 acceleration vectors measured when each axis of the accelerometer is pointed vertically upward and downward. For example, $\boldsymbol{a}_{(x,+)}^S$ and $\boldsymbol{a}_{(x,-)}^S$ represent the acceleration vectors measured when the accelerometer is pointed vertically upward and downward along the X^S axis, respectively. Similarly measuring the Y^S and Z_S axis, the 6 acceleration vectors use to generate the pseudo–accelerations vector are $\{\boldsymbol{a}_{(x,+)}^S, \boldsymbol{a}_{(x,-)}^S, \boldsymbol{a}_{(y,+)}^S, \boldsymbol{a}_{(y,-)}^S, \boldsymbol{a}_{(z,+)}^S, \boldsymbol{a}_{(z,-)}^S\}$. As an example, when generating pseudo data for a space where the X^S, Y^S, and Z^S axes are all in the $+$ direction as shown in Fig. 2, using the acceleration vectors $\{\boldsymbol{a}_{(x,+)}^S, \boldsymbol{a}_{(y,+)}^S, \boldsymbol{a}_{(z,+)}^S\}$, it is expressed as

$$\boldsymbol{a}^P(\theta, \phi) = (\boldsymbol{a}_{(x,+)}^S \cos\theta + \boldsymbol{a}_{(y,+)}^S \sin\theta) \cos\phi + \boldsymbol{a}_{(z,+)}^S \sin\phi. \tag{7}$$

Note that $\theta = \phi = 0 \sim \frac{\pi}{2}$. In this way, X^S, Y^S, and Z^S axes are selected from the 6 measured acceleration vector and substituted into (7) to generate the pseudo–acceleration vector when the accelerometer is pointed in any direction for 360°.

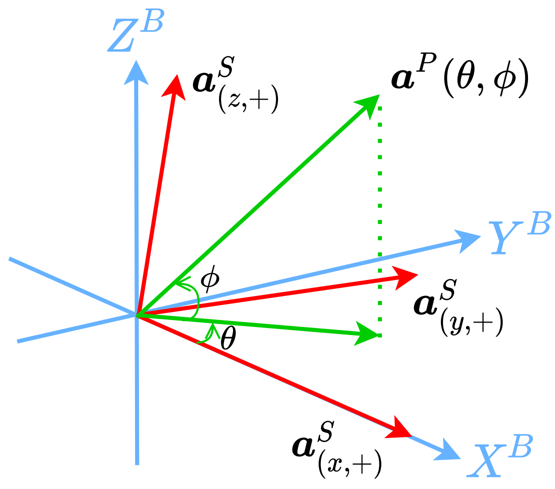

Fig. 2. Generating the pseudo–acceleration vector.

III. Analysis and Discussion

In this chapter, by considering the difference between the pseudo–acceleration vector and the observed acceleration vector, we will discuss the influence of the pseudo–acceleration vector on the parameter estimation results.

A. Expressions for generating pseudo–acceleration vector

The ideal acceleration vector $\boldsymbol{a}^O$ for a space in which the X^S, Y^S, and Z^S axes are all in the $+$ direction, unaffected by misalignment, scale factor, and zero–g bias, can be calculated using the local gravity g:

$$\begin{aligned}\boldsymbol{a}^O &= (\boldsymbol{g}_{(x,+)}\cos\theta+\boldsymbol{g}_{(y,+)}\sin\theta)\cos\phi+\boldsymbol{g}_{(z,+)}\sin\phi\\ &= \left\{\begin{bmatrix} g\\0\\0\end{bmatrix}\cos\theta+\begin{bmatrix}0\\g\\0\end{bmatrix}\sin\theta\right\}\cos\phi+\begin{bmatrix}0\\0\\g\end{bmatrix}\sin\phi.\end{aligned} \tag{8}$$

Then, when $\boldsymbol{a}^O$ is affected by misalignment, scale factor and zero–g bias, the acceleration vector $\boldsymbol{a}^S$ can be calculated from (1):

$$\boldsymbol{a}^S=(\boldsymbol{T}^a\boldsymbol{K}^a)^{-1}\boldsymbol{a}^O-\boldsymbol{b}^a. \tag{9}$$

Looking at (9), we see that the zero–g bias vector $\boldsymbol{b}^a$ is always constant, and is independent of the direction of the ideal acceleration vector $\boldsymbol{a}^O$.

Now, the observed acceleration vector used for the calibration method described in the previous chapter was the 6 acceleration vector measured with each axis pointing vertically upward and downward. This observed acceleration vectors $\{\boldsymbol{a}^S_{(x,+)},\boldsymbol{a}^S_{(x,-)},\boldsymbol{a}^S_{(y,+)},\boldsymbol{a}^S_{(y,-)},\boldsymbol{a}^S_{(z,+)},\boldsymbol{a}^S_{(z,-)}\}$ are generated from (9). And then, substituting (9) into (7) and rearranging it, the pseudo–acceleration vector when the X^S, Y^S, and Z^S axes are all in the $+$ direction can be expressed as follows:

$$\begin{aligned}&\boldsymbol{a}^P(\theta,\phi)\\ &= (\boldsymbol{a}^S_{(x,+)}\cos\theta+\boldsymbol{a}^S_{(y,+)}\sin\theta)\cos\phi+\boldsymbol{a}^S_{(z,+)}\sin\phi\\ &= (\boldsymbol{T}^a\boldsymbol{K}^a)^{-1}\\ &\quad\left\{(\boldsymbol{g}_{(x,+)}\cos\theta+\boldsymbol{g}_{(y,+)}\sin\theta)\cos\phi+\boldsymbol{g}_{(z,+)}\sin\phi\right\}\\ &\qquad -\boldsymbol{b}^a\left\{(\cos\theta+\sin\theta)\cos\phi+\sin\phi\right\}\\ &= (\boldsymbol{T}^a\boldsymbol{K}^a)^{-1}\boldsymbol{a}^O-\boldsymbol{b}^a\left\{(\cos\theta+\sin\theta)\cos\phi+\sin\phi\right\}\end{aligned} \tag{10}$$

From (10), the first term agrees with (9), therefore the generated pseudo–acceleration vector is equivalent to the observed acceleration vector even if the angles θ,ϕ are arbitrarily set. On the other hand, the zero–g bias vector $\boldsymbol{b}^a$ in the second term depend on the angle θ and ϕ given when generating the pseudo vector. Cconsequently, depending on how the angle θ,ϕ is given, there is a possibility of generating the pseudo–acceleration vector that does not reflect the zero–g bias vector, hence care must be taken.

TABLE I
Set values of the nine correction parameters in the simulation.

Error factor	Variable	Value
misalignment	α_{yz}	0.0049
	α_{zy}	-0.0055
	α_{zx}	0.0079
scale factor	s_x	0.9908
	s_y	1.0068
	s_z	1.0066
zero–g bias	b_x	0.0793
	b_y	-0.0024
	b_z	0.0636

B. Number of pseudo–acceleration vector used for calibration and accuracy of parameter estimation

In this section, we evaluated the accuracy of parameter estimation against the number of the pseudo–acceleration vector. Table I is the parameters to be calibrated set in this simulation. The observed acceleration vectors were generated from (9) using the parameters in Table I, and used to estimate the correction parameters and to generate the pseudo–acceleration vector. The number of the acceleration vector was varied up to $6\sim 30$, and the angles θ,ϕ were set randomly. The correction parameters were estimated following the procedure described in Chapter 2. The estimation accuracy was evaluated as the standard deviation of the difference between the true and estimated parameters shown in Table I after 30 trials of parameter estimation.

Fig.3 shows the estimation accuracy of the correction parameters estimated using only the observed acceleration vectors (i.e., the correction parameters estimated using the method in [4]). The figure shows that the estimation error of the correction parameters decreases as the number of observation vectors increases.

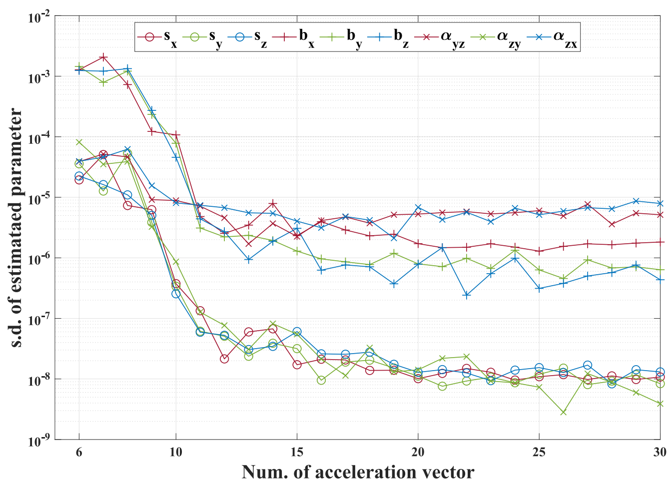

Fig. 3. Parameter estimation accuracy evaluation for each number of observed acceleration vector.

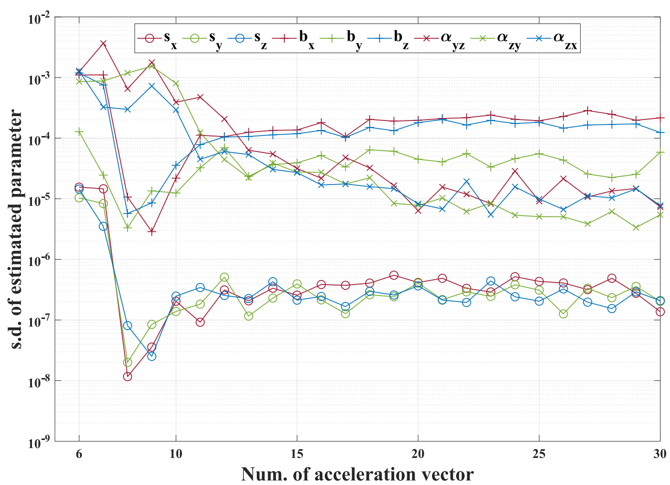

Fig. 4. Parameter estimation accuracy evaluation for each number of pseudo–acceleration vector.

On the other hand, Fig.4 shows the estimation accuracy of the correction parameters estimated using blended the pseudo–acceleration and 6 observed acceleration vectors. First, the estimation accuracy of misalignment is improved by increasing the number of postures. In particular, for misalignments α_{yz} and α_{zx}, there is no difference in estimation accuracy compared to estimation using only observation vectors. Second, the estimation accuracy of the scale factor and zero–g bias decreased as the number of postures increased. This may be due to the fact that the estimation was performed using the pseudo–acceleration vector that did not reflect the zero–g bias vector analyzed in the previous section.

IV. Conclusion

This paper showed how the pseudo–acceleration vector affects the calibration algorithm. The analysis results suggest that the calibration accuracy may be affected by the method of generating the pseudo–acceleration vector. In future work, we would like to study the method of generating pseudo data considering the zero–g bias vector.

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