Design and comparison of passive and active control methods in absorbing torsional vibrations of a vertical drilling string with time delay

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***Abstract*— In oil drilling systems, vibrations are one of the main sources of economic losses. In this paper, a non-linear continuous model with time delay for the vertical drilling string is proposed to study the torsional vibrations of the drilling string. The Karnopp friction model is used to simulate the stick-slip phenomenon in the interaction between the ground and the drill. In the following, two different methods for reducing torsional vibrations and controlling the stick-slip phenomenon of the drilling string are investigated. In the first method, the effect of changing the system parameters on reducing the torsional vibrations is investigated. In the second method, the torsional vibrations of the drill string are controlled using the pole placement method. In this way, corresponding to the desired response of the system, with the search of parameters space, the appropriate location of the closed-loop poles of the system is found. With the designed control system, it is observed that the torsional vibrations are absorbed and the stick-slip phenomenon is damped.**

Keywords— drill string, torsional vibrations, stick-slip phenomenon, non-linear continuous model with time delay, passive & active vibration control

I. INTRODUCTION

Vibrations in mechanical systems cause undesirable oscillations. These vibrations have adverse effects on system components and may cause premature failure. In oil drilling systems, vibrations are one of the main sources of economic losses. Drilling failure, disconnection between oil pipes, reduction of component life, an increase of drilling time, etc. are just some of the harmful consequences of vibrations in drilling systems. These vibrations also reduce efficiency, improper use of energy, instability of wells, and their deformation. At present, the absorption of drilling vibrations has opened a new horizon for engineers to absorb these vibrations, reduce the amount of damage, drilling time, and drilling costs and increase the productivity.

Several studies have been carried out to detect different types of vibrations in drill strings, which has led to the identification and classification of these vibrations into three main categories: torsional, axial, and lateral vibrations. Angular drill speed measurements show that using a constant rotational speed on the surface does not cause the drill to rotate continuously. The rotational speed of the drilling bit varies considerably with amplitude. The stick-slip phenomenon occurs when the drill gets stuck in the well and does not rotate, while the motor is running at a constant speed. When the trapped energy reaches a point that the drill is unable to withstand, the energy is suddenly released and the drill begins to rotate at high speed. The behavior of the stick-slip phenomenon is like a torsional wave that moves along the drilling string.

Many efforts have been made in modeling and controlling drilling systems. Before the 1960s, more research focused on the material and strength of drilling bit. In 1960, Bailey [1] conducted the first experimental and analytical studies on the axial and torsional vibrations of the drill string. After that, various approaches have been proposed. Hale and Huang [2] controlled the waves generated along the drill string by the Top extremity by adjusting the rotational speed of the drill bit according to the torque applied to it. Jansen and Steen [3] have proposed an improved method of torque feedback system in which instead of measuring torque, the torque is measured by measuring the motor current. Pavković et al. [4] have proposed a way to prevent the drill string from slipping using a PID controller. Lopez [5] has presented a sliding mode control based on the analysis of system parameters with lumped mass modeling. Petrovskii et al [6] have developed a semiactive vibration damping method for adaptive control of drill string vibrations. Lu et al [7] have developed an advanced control system to mitigate torsional vibration of drill-string using a measurement-while-drilling. Cheng et al [8] have proposed a systematic linear parameter varying (LPV) model and a gain-scheduled control methodology for drill-string systems and suppress stick-slip vibrations. Fu et al [9] have proposed a control strategy based on a state observer and a reference governor to suppress stick-slip vibrations of the drill-string. Vaziri et al [10] have investigated experimentally and numerically suppression of drill-string torsional vibration using a sliding mode control.

Dynamic modeling of the drill string is the basis of system analysis and control system design to absorb destructive vibrations. Despite the development of various methods to reduce the vibrations of the drill string, today, this phenomenon greatly affects the drilling processes. The main reason for the poor performance of vibration absorption methods is a misunderstanding of system dynamics. Most of the models are based on simplified dynamics in which the mass distribution, system parameters as well as the existence of time delay in the system are ignored.

In this paper, a non-linear continuous model with time delay for the vertical drilling string is proposed to study the torsional vibrations of the drilling string. Then, an active and passive method is proposed for the reduction of torsional vibrations of the drill string.

II. Dynamic Modelling of the Vertical Drill String through Equations of Motions

A. Modeling

The propagation of a torsional wave along the drill string of length L is expressed by the following differential equation [11]:

$$GJ\frac{\partial^2 \phi}{\partial s^2}(s,t) - I\frac{\partial^2 \phi}{\partial t^2}(s,t) - \gamma\frac{\partial \phi}{\partial t}(s,t) = 0 \ , \quad s \in (0,L) \qquad (1)$$

where ϕ is the twist angle of the drill string, which depends on the position s and time t. G and J show the shear modulus and the second moment of area, respectively. Also, γ is the viscous damping coefficient. Because most of the energy dissipation in drilling systems is taking place at the bit-rock interface, damping is ignored, which leads to:

$$\frac{\partial^2 \phi}{\partial s^2}(s,t) = \tilde{c}^2\frac{\partial^2 \phi}{\partial t^2}(s,t) \qquad (2)$$

where $\tilde{c} = \sqrt{\frac{I}{GJ}}$ is the wave propagation speed.

B. Boundary conditions

Eq. (3) presents the boundary conditions according to the Newton's second law:

$$I_T\frac{\partial^2 \phi}{\partial t^2}(0,t) = -T_T(t) + u_T(t) \qquad (3)$$

where I_T and u_T show the effective moment of inertia of the top-drive and the external torque delivered by the rotary table taken as a control input, respectively. T_T describing the transmitted torque and damping due to viscous effects, is given by:

$$T_T(t) = -GJ\frac{\partial \phi}{\partial s}(0,t) + \beta\frac{\partial \phi}{\partial t}(0,t) \qquad (4)$$

The boundary conditions are then written as:

$$GJ\frac{\partial \phi}{\partial s}(0,t) = I_T\frac{\partial^2 \phi}{\partial t^2}(0,t) + \beta\frac{\partial \phi}{\partial t}(0,t) - u_T(t) \qquad (5)$$

$$GJ\frac{\partial \phi}{\partial s}(L,t) = -I_B\frac{\partial^2 \phi}{\partial t^2}(L,t) - T\left(\frac{\partial \phi}{\partial t}(L,t)\right) \qquad (6)$$

C. Time delay

The wave equation model provides a description of the distributed system variables; however, due to the relatively long length of the drill string and the movement of the wave along the drill string, the system has a time delay. Measurements show that the angular velocity of the end of the drill in the present depends not only on its value in the past, but also on the rate of change in the past. This type of time delay in the system can be modeled using the D'Alembert method.

Assuming the variables $\gamma = t + \tilde{c}t$ and $\eta = t - \tilde{c}t$ the solution of the undamped wave equation (1), the torsional drill string behavior, is given by:

$$\phi(s,t) = \varphi(\gamma) + \psi(\eta) \qquad (7)$$

By substituting Eq. (7) and the boundary conditions (5) and (6) in Eq. (1), the ordinary differential equation of the following is obtained:

$$\begin{aligned}\ddot{\phi}_b(t) - \Upsilon\ddot{\phi}_b(t-2\tau) &= -\psi\dot{\phi}_b(t) - \Upsilon\psi\dot{\phi}_b(t-2\tau) \\ &\quad - \frac{1}{I_B}T\left(\dot{\phi}_b(t)\right) + \frac{1}{I_B}\Upsilon T\left(\dot{\phi}_b(t-2\tau)\right) \\ &\quad + \Pi_\Omega u_T(t-\tau) \end{aligned} \qquad (8)$$

where φ_b is the angular position at the bottom extremity, and:

$$\Pi_\Omega = \frac{2\psi}{\beta + \tilde{c}GJ}\ , \quad \Upsilon = \frac{\beta - \tilde{c}GJ}{\beta + \tilde{c}GJ}\ , \quad \psi = \frac{\tilde{c}GJ}{I_B}\ , \quad \tau = \tilde{c}L \qquad (9)$$

D. Friction Modeling

Instabilities are caused by the interaction between the ground and the drill, and their growth leads to increased torsional and axial oscillations. An appropriate friction model makes a correct understanding of the system behavior and a suitable control strategy to absorb vibrations is designed [11].

To model the friction between the drill and the ground, the Karnopp's model with an exponential decaying friction term is used [5].

$$f_b\left(\dot{\phi}_b(t)\right) = W_{ob}R_b\mu_b\left(\dot{\phi}_b(t)\right) \qquad (10)$$

$$\mu_b\left(\dot{\phi}_b(t)\right) = \mu_{cb} + (\mu_{sb} - \mu_{cb})e^{-\gamma_b|\dot{\phi}_b(t)|} \qquad (11)$$

III. Stability Analysis of the Problem

In this section, the stability of the system is discussed using the root locus method. Because the model is solved numerically, a Fourier series corresponding to the numerical solution of the model is obtained. Then, the approximate transfer function of the system is calculated for the angular velocity of the drill bit.

A. Vibrations simulation

Parameters of the system are used from the Shell Co. for a typical drill string [12]. Table (1) shows the value of the parameters. Fig. (2) shows the numerical solution of the model.

TABLE1 Value of the System Parameters

Parameter	*Numerical value*	*Parameter*	*Numerical value*
String length	1172 m	Inertia	0.095 Kg m
Shear modulus	79.3×10^9 N m^{-2}	Angular momentum	2000 N m s
Young modulus	200×10^9 N m^{-2}	Viscous friction coefficient	200.025 Kg s^{-1}
Drillstring's cross-section	35×10^{-4} m2	Damping	16100 Kg s^{-1}
Second moment of area	1.19×10^{-5} m^4	Torsional stiffness	473 N m rad^{-1}

The best Fourier series for the angular speed of the bit is:

$$\begin{aligned}\omega(t) = 9.508 &+ \{-11.83\,cos(2.367t) \\ &- 6.647\,sin\,2.367(t)\} \\ &+ \{-0.0143cos(4.734t) \\ &+ 1.018\,sin(4.734t)\} \\ &+ \{-0.2319cos(7.101t) \\ &- 0.3199\,sin(7.101t)\}\end{aligned} \qquad (12)$$

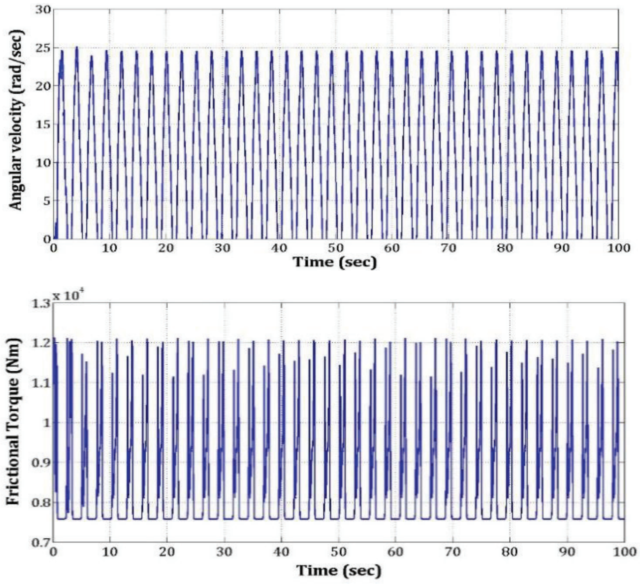

Fig. 1. Simulation of torsional vibrations for Ω_0 = 12.5 rad / sec (*angular speed of drill bit and friction torque*)

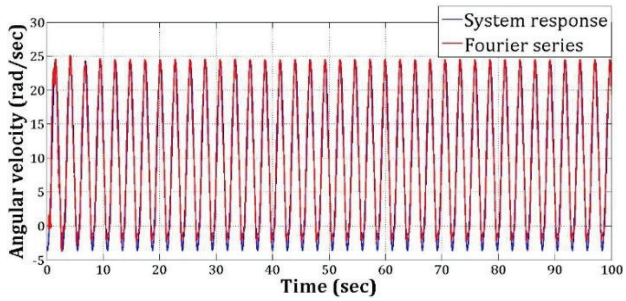

Fig. 2. System response and Fourier approximation for the angular velocity of the bit

As seen in fig. (2), the Fourier approximation best fits with the system response.

B. Stability analysis

The root locus diagram is shown in Fig. (3). The system is unstable for all values of a simple proportional controller with the gain k.

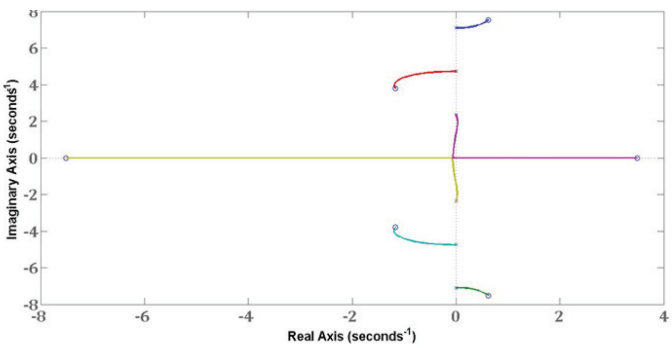

Fig. 3. Root locus diagram of the system

IV. Reduction of Torsional Vibrations Using Changes in System Parameters

A. Reducing the weight on bit[1]

For a certain angular velocity of the motor, the amount of torsional vibration is reduced by reducing WoB. As shown in Fig. (4), torsional vibrations are reduced by decreasing the WoB from 97kN to 40kN.

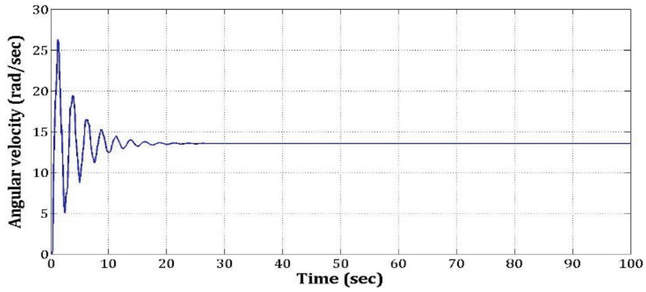

Fig. 4. Simulation of torsional vibrations for WoB=40kN

B. Increase the angular velocity of the motor

Increasing the angular velocity of the motor reduces the torsional vibrations, as shown in Fig. (5).

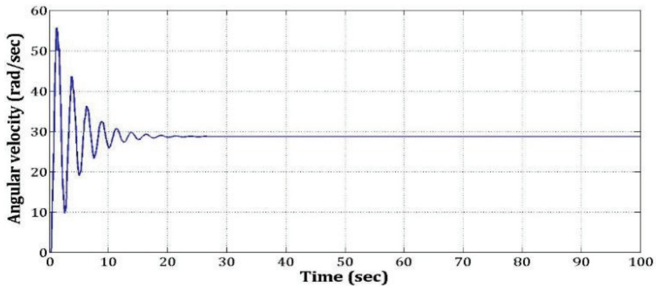

Fig. 5. Simulation of torsional vibrations for Ω_0=30 rad / sec

V. Controller Design

A. State space

State space equations are given through the following equations:

$$\dot{\mathbf{x}} = A\mathbf{x} + Bu\,, \qquad y = C\mathbf{x} + Du \tag{13}$$

in which,

$$A = \begin{bmatrix} 0 & -78.4 & 0 & -1538 & 0 & -6332 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$C = [13 \quad -78 \quad 940 \quad -5173 \quad 1670 \quad -76461], D = [2.568]$

Because the following controllability matrix is full rank, the system is full state controllable.

$$M_c = [B|AB|A^2B|\ldots|A^5B] \tag{14}$$

B. Controller design using pole placement method

Desired closed-loop poles are obtained according to the desired system response (overshoot and settling time). The control law is defined as:

$$u = -K\mathbf{x} \tag{15}$$

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[1] WoB

$$\dot{\mathbf{x}} = (A - BK)\mathbf{x} \quad , \quad y = (C - DK)\mathbf{x} \tag{16}$$

For determining controller matrix K, Ackermann's method is used [13]:

$$K = [0\ 0\ 0\ 0\ 0\ 1]M_c^{-1}\Delta(\mathbf{A}) \tag{17}$$

where M_c is the controllability matrix and $\Delta(A)$ defined as bellow:

$$\Delta(\mathbf{A}) = \mathbf{A}^6 + \alpha_1\mathbf{A}^5 + \cdots + \alpha_5\mathbf{A} + \alpha_6\mathbf{I} \neq \mathbf{0} \tag{18}$$

in which α_i are the coefficients of the characteristic equation of the system based on desired closed-loop poles (μ_i):

$$(s - \mu_1) \ldots (s - \mu_6) = s^6 + \alpha_1 s^5 + \cdots + \alpha_5 s + \alpha_6 \tag{19}$$

VI. Simulation and Results

In the previous section, the control signal corresponding to each controller was obtained using the control law relation. Then the behavior of the system is studied by applying the controllers on the model and numerical solution of the model. The response of the model comes after applying controllers designed for different values of damping ratio and natural frequency. By determining the required values for the overshoot and settling time, the desired closed-loop poles can be calculated. Based on the desired poles, the corresponding controller is designed. after applying the controller to the system, it is observed that the desired response is obtained (figs. (6-8)). The response of the system for the three different modes of settling time of fewer than 10 seconds, 20 seconds, and 50 seconds and overshoot 5% and 10% is shown in the following figures. Due to the time delay in the original model, after applying the control signal, the system responds with a delay, which is shown in the figures.

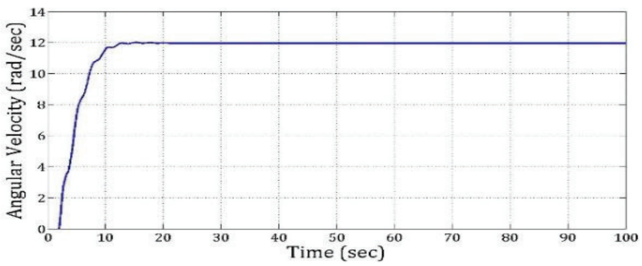

Fig. 6. System response for settling time<10 sec and overshoot<5%

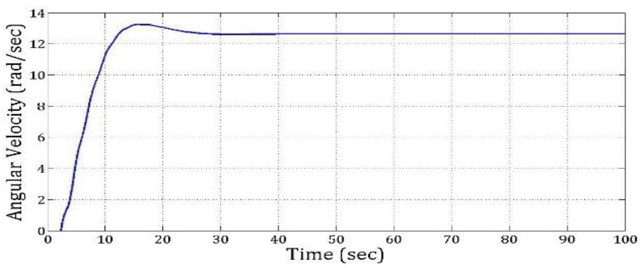

Fig. 7. System response for settling time<20 sec and overshoot<10%

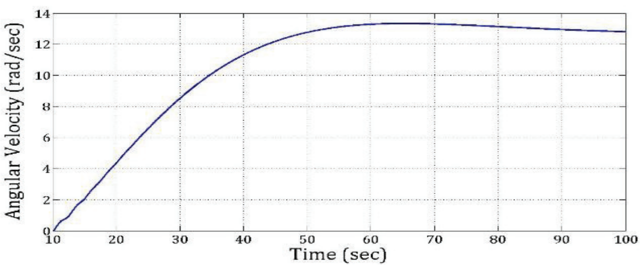

Fig. 8. System response for settling time<50 sec and overshoot<10%

VII. Conclusions

In this paper, a continuous non-linear model with time delay is proposed for torsional vibrations in the drill string. Then, two different methods to reduce torsional vibrations and control the stick-slip phenomenon of the drill string were investigated. In the first method, the effect of changing system parameters on reducing the torsional vibrations was studied. It can be seen that by increasing the angular speed and decreasing WoB, torsional vibrations can be significantly reduced. In the second method, the system state space equations were obtained. Then the torsional vibrations of the drill string were controlled using the pole placement method. In this way, corresponding to the optimal response of the system, by searching the space of parameters, the appropriate location of the closed loop poles of the system was obtained. By applying new conditions to the control system, it is observed that torsional vibrations are partially absorbed and the stick-slip phenomenon is inhibited. The pole placement method reduces vibrations by changing the pattern of the control signal (motor torque). It can be seen that as the settling time increases, the applied torque does not increase, but the torque reaches its maximum value. Also, with increasing settling time and due to time delay in the system, the drilling bit starts to rotate later. Due to the limitations in presentation, qualitative and quantitative comparisons can be studied as future work.

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