The First Experimental Evidence of Chaos from a Nonlinear Circuit with a Real Memristor

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***Abstract* — In this work, chaos is experimentally observed from a circuit, in which the nonlinear element is a real memristor. To the best of our knowledge, this is the first nonlinear circuit with a commercially available memristor (KNOWM memristor), which has been implemented in order to experimentally investigate chaos and phenomena related with it, such as a route to chaos via period-doubling, one-scroll and the well-known double-scroll chaotic attractors.**

Keywords— Chaos, double-scroll attractors, KNOWM Memristor, nonlinear circuit

I. Introduction

Chaos is the phenomenon, which has interesting applications in many research fields, such as chemistry, biology, ecology, physics, economics, sociology [1-6], etc. Especially, in engineering, systems with chaotic behavior have found interesting and significant applications in electronics, control, communication, cryptography and robotics [7-10].

From all the aforementioned research fields, the field of design nonlinear circuits attracts the interest of the researchers more, due to the easy way that the chaotic phenomena can be simulated with them, as well as due to their big number of chaos-based applications. Design of chaotic secure communication schemes, chaotic cryptography, robotics and design of nonlinear neuronal networks are some of them [11-15]. Furthermore, the design and implementation of new circuits with nonlinear elements is a fascinating subject with many research challenges to overcome.

Important role in the evolution of the research field of nonlinear circuits, the last five decades, has played Professor Leon O. Chua with the invention of Chua's circuit and the memristor (a word blending of memory and resistor), which is known as the fourth fundamental circuit element.

The first of these inventions, i.e. the Chua's circuit [16], is the first nonlinear circuit, which was designed by Prof. Chua, in 1983 and was constructed by his research team for displaying chaos. His second invention, which was chronologically the first one, is the memristor [17], which is a circuit element that presents the missing link between magnetic flux and electric charge. The concept of memristor has been presented in a seminal paper by Prof. Chua in 1971. Since then, only few works had been reported in literature for a long time. The main reason for this was that the new proposed circuit element was only a theoretical concept. So, until 2008, memristor had received little attention. However, that year, researchers in Hewlett-Packard published an article, in which a physical model of memristor was presented [18]. In the proposed model the memory effect was achieved in a solid-state thin film two-terminal device fabricated by titanium dioxide TiO_2 sandwiched between platinum electrodes.

Since then, a great number of articles have been published presenting different models of memristors, design techniques and fabricated materials [19-24], as well as important applications of memristors, such as in associative memory [25], neural networks [26], adaptive filters [27], high-speed low-power processors [28], programmable analog integrated circuits [29], Boolean logic gates [30], and so on.

One of the most important feature of memristor is the nonlinear relationship between current and voltage. This inherent feature of memristor could be exploited in the direction of design new chaotic systems with complex dynamics. Also, memristor can replace well-known nonlinear elements, such as Chua's diode, in circuits [31-33], or nonlinear functions in conventional dynamical systems, [34,35], in order to design new ones with advanced features.

Until now there is no published work on designing circuits for emulating chaotic systems, in which a real memristor is used as a nonlinear element, due to the lack of reliable commercially available memristors. Researchers have designed and realized only memristor emulators for use in nonlinear chaotic circuits. However, in the last two years memristors from the KNOWM Inc. are commercially available [36, 37].

In this work, for the first time, as far as the authors know, an experimental demonstration of a commercially available memristor, i.e. KNOWM memristor, in a well-known nonlinear circuit, implementing a 3D dynamical system, in order to produce chaos, is presented.

The remainder of this work is organized as follows. In the next Section, the analysis of the memristor-based nonlinear circuit, as well as the details of its realization, are presented. In Section III the experimental results of the circuit's behavior, for various values of its control element are presented. Finally, the last Section draws the concluding remarks and some thoughts for future work.

II. THE PROPOSED NONLINEAR CIRCUIT

The memristor-based chaotic oscillator, which is used in this work, it is based on a well-known Shinriki's circuit, introduced in 1981. This circuit is a modification of the well-known van der Pol circuit and it consists of a parallel resonant circuit (R_5LC_2), which is coupled to a R_4C_1 linear oscillator through a nonlinear positive conductance. The Shinriki's circuit has also a negative impedance converter. This converter consists of an operational amplifier, two identical resistors R_1 and R_2, and a resistor R_3 connected to the ground. In this way the negative impedance converter acts as a linear negative resistor $-R_3$. Shinriki *et al*., presented in their work that the proposed circuit can generate periodic waveforms or random-like waveforms depending on the chosen parameters (for more information the reader can read Ref. [38]).

In 1984 Freire *et al*. presented further results from the investigation of the Shinriki's circuit [39]. They discovered a great variety of dynamical behaviors that the circuit produced, such as equilibrium points, periodic and chaotic attractors, as well as various types of bifurcations, i.e. Hopf and pitchfork bifurcations, flip bifurcations, etc. This study pointed out the interest devoted to the aforementioned circuit.

Furthermore, in 2015 a novel autonomous memristor-based nonlinear circuit, which also belong to the well-known Chua's circuit family and presenting also interesting phenomena related to chaos, is introduced [40]. This circuit is obtained from Shinriki's circuit, by using a memristive diode bridge, implemented by using a first order parallel *RC* filter which replaces the nonlinear positive conductance.

In this work the commercial memristor device, manufactured by KNOWM Inc., has been used in the Shinriki's circuit instead of the nonlinear positive conductance. So, the schematic of the circuit, which has been used, is presented in Fig. 1, while the real circuit, which has been constructed, is shown in Fig. 2. In this circuit variable resistor R_5 and inductor L, have been used. Also, the user, through a switch of eight positions, can choose the one of eight discrete Self-Directed-Channel (SDC) memristors from the 16-pin ceramic DIP package of KNOWM device.

The current-voltage characteristic curve of the second KNOWM memristor in the 16-pin ceramic DIP package, which has been used in the circuit of Fig. 2, is depicted in Fig. 3. This *i-v* curve has been captured by using the Analog Discovery 2 USB oscilloscope. As it is experimentally verified the KNOWM device presents the three characteristic fingerprints for identifying a device as a memristor [41].

The nonlinear circuit of Fig. 1 is mathematically described by the set of differential equations:

$$\begin{cases} C_1 \dfrac{dv_1}{dt} = \left(\dfrac{1}{R_3} - \dfrac{1}{R_4}\right) v_1 + i_M \\ C_2 \dfrac{dv_2}{dt} = -i_L - \dfrac{v_2}{R_5} - i_M \\ L \dfrac{di_L}{dt} = v_2 \end{cases} \tag{1}$$

where i_M represents the input current through the memristor. So, the equilibria of the proposed system are $(v_1, v_2, i_L) = (v_1^*, 0, kv_1^*)$, where $k = \dfrac{1}{R_3} - \dfrac{1}{R_4}$. Therefore, the nonlinear circuit of Fig. 1 is a simple 3D nonlinear dynamical system, in which the only nonlinear term is the input current i_M through the memristor, and also it has an infinite number of equilibrium points located in the line $i_L = kv_1$.

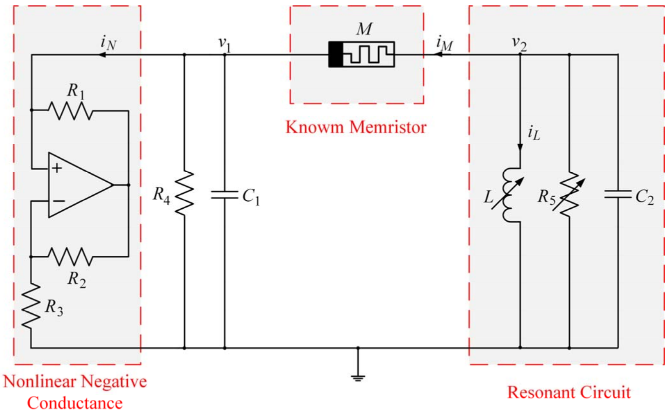


Fig. 1. The schematic of the proposed memristor-based nonlinear circuit.

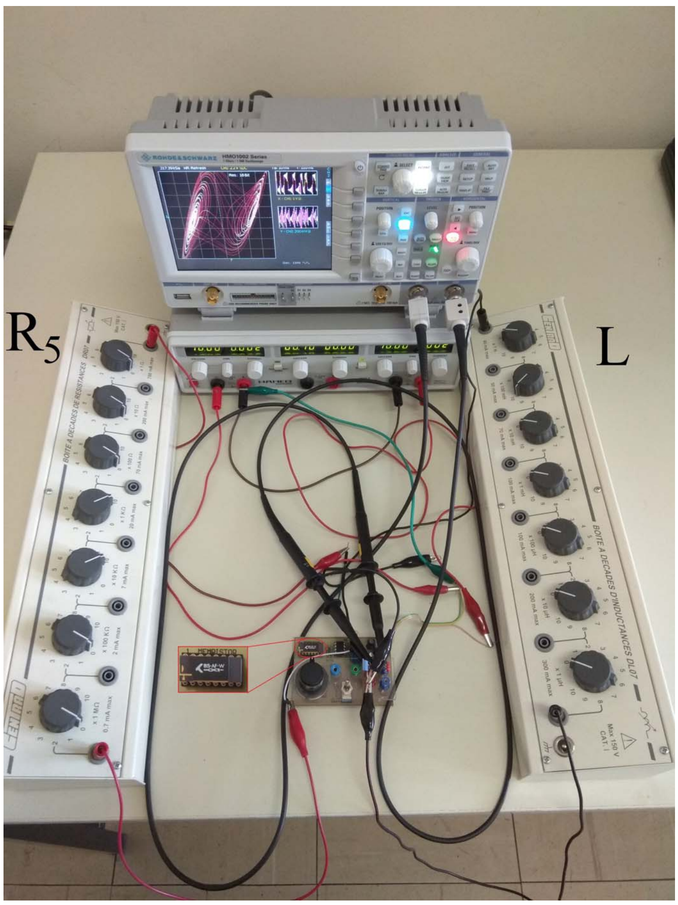


Fig. 2. Experimental setup of the circuit with the board with KNOWM memristor chip, the variable resistor R_5 and inductor L.

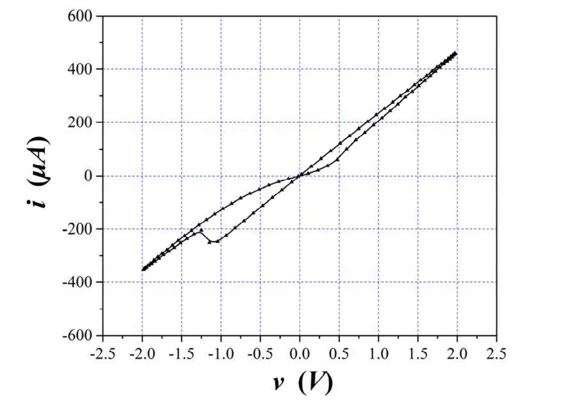


Fig. 3. The *i*-*v* characteristic curves of the second KNOWM memristor with voltage amplitude 2 V and frequency 500 Hz.

III. Experimental Results

In this work, the challenge of building a circuit, which has as a nonlinear element a real memristor, in order to experimentally observe chaos, is faced. Due to the fact that the resonant frequency ω is determined by the value of the inductor L according to equation

$$\omega = \frac{1}{\sqrt{LC_2}} \tag{2}$$

the shape of the current-voltage characteristic curve of the memristor is not always the same.

It is well known from theory of memristor that at high frequencies, one of its characteristic fingerprints, which is the appearance of pinched hysteresis loop, will be driven to its degeneration, resulting in a diagonal line corresponding to a linear resistance. Also, the *i*-*v* characteristic curve of a real memristor, like this, is not as stable as the respected *i*-*v* characteristic curves produced based on mathematical models in simulation. Furthermore, as observed from the manufacturer's datasheet [42] and from the authors' experimental work, in which the aforementioned Knowm chip with the eight memristors has been used, the pinched hysteresis loops provided by these memristors are very distorted and far from ideal (Fig. 3). Another drawback is the unstable behavior of the KNOWM device leading the memristor to lose the *i*-*v* characteristic curve and drives the circuit to saturation. These are the main problems for many researchers who want to use a real memristor like this in nonlinear circuits.

However, the advantage of this memristor is that its pinched hysteresis loop, which is the basic characteristic of this device, is kept for a wide range of frequencies and voltage amplitude values (around ±3.0 V) by changing only its shape. So, it is capable of using it in analog nonlinear circuits for observing phenomena related to chaos theory.

So, by using the circuit of Fig. 2, its experimental dynamical behavior is observed with the following values of circuit's elements: C_1 = 10 nF, C_2 = 101.4 nF, R_1 = R_2 = 5.6 kΩ, R_3 = 10 kΩ, R_4 = 60 kΩ, R_5 = 100 kΩ, op-amp LF411, while the power supply is ±10 V. For these values of circuit's elements, the route to chaos through a period doubling cascade is experimentally observed as the tunable inductor L is slowly increased (Fig. 4a-4c). Also, one-scroll chaotic attractor (Fig. 4d), period-3 attractor (Fig. 4e) and the well-known double-scroll chaotic attractor (Fig. 4f), similar to the attractors produced from Chua's circuits family, are revealed, as the control parameter (L) increases. Fig. 5 depicts the time-series of signals v_1 and v_2, in the case of a double-scroll attractor.

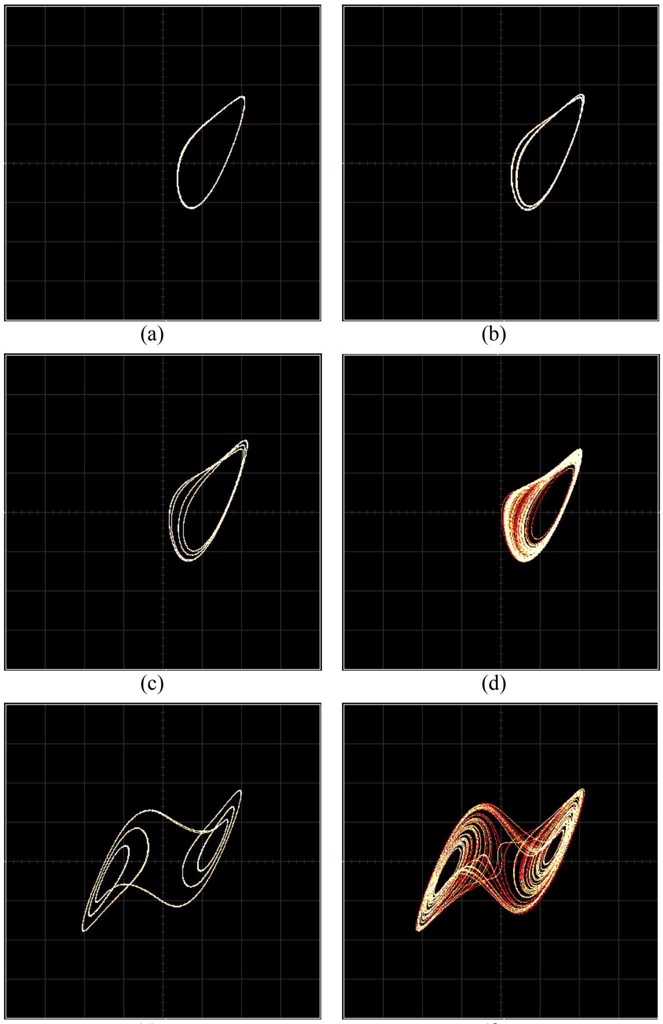
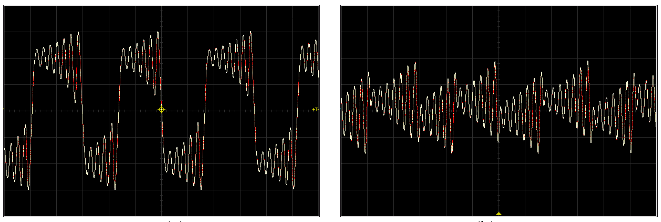


Fig. 4. Experimental attractors in v_2 vs. v_1 plane, for R_5 = 100 kΩ. (a) L = 0.73 H (period-1 attractor), (b) L = 0.76 H (period-2 attractor), (c) L = 0.79 H (period-4 attractor), (d) L = 0.80 H (one-scroll chaotic attractor), (e) L = 0.90 H (period-3 attractor), (f) L = 1.0 H (double-scroll attractor). X: 1 V/DIV, Y: 0.2 V/DIV.

IV. Conclusion

In this paper, the experimental investigation of a nonlinear circuit's dynamics, in which the nonlinear element has been replaced with a real memristor, was presented for the first time, as far as the authors know. Despite the drawbacks of the KNOWM memristor device, which was used, concerning especially the loss of the *i*-*v* characteristic curve in some cases after changing the values of circuit's elements, as well as the fact that its pinched hysteresis loops are highly distorted and far from ideal, the circuit with the memristor worked very satisfactory. As a consequence the experimental observation of chaos produced by the proposed circuit for various cases of



(a) (b)

Fig. 5. Time-series of signals (a) v_1 (X: 0.5 V/DIV) and (b) v_2 (X: 0.2 V/DIV), in the case of R_5 = 100 kΩ and L = 1 H (10 ms/DIV).

inductor's values was presented. Furthermore, phenomena concerning the theory of chaos, such as one-scroll and double-scroll chaotic attractors, as well as a period-doubling route to chaos, were experimentally observed, presenting similar behavior as the circuits of Chua's family. As future plans, the use of other or the same real memristors in known or novel circuits and discovering the range of their dynamics and the possible real-world applications of such circuits, have been planned.

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