Pareto-Optimal Resource Allocation in Wireless Powered Networks

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***Abstract*—In this paper, the uplink of a wireless-powered network is investigated. More specifically, we focus on simultaneously maximizing the sum-throughput and minimum throughput of the network's users, for the case of non-orthogonal multiple access (NOMA) and time-division multiple access (TDMA), by optimizing the allocated time to energy harvesting and information transmission. Since this problem belongs to the category of multi-objective optimization, we transform it into a single-objective problem, via the scalarization approach, aiming to obtain the Pareto Front. The proposed methodology facilitates the evaluation of the trade-off between the considered conflicted metrics. Finally, simulation results validate the effectiveness of the proposed methodology and provide useful insights for the network's performance.**

***Index Terms*—non-orthogonal multiple access (NOMA), wireless powered networks (WPNs), multi-objective optimization (MOO), Pareto front**

I. Introduction

The limited life of devices' batteries, can be a significant limitation in communication networks' equipment. By recharging the batteries through radio frequency (RF) signals, the problem of their substitution is solved, which can be costly or even infeasible, [1]. Furthermore, the received energy could also be used for communication purposes, i.e., devices are able to transmit data to the base station (BS), which is the main principle of wireless powered networks (WPNs). In these networks, users adopt the *harvest-then-transmit* protocol, where in the first phase the BS transmits energy to the devices, while in the second phase, this energy is used for information transmission [2].

Currently, a new promising multiple access scheme, namely non-orthogonal multiple access (NOMA), is a strong candidate for usage in the fifth generation (5G) of wireless networks and beyond. The fundamental difference of NOMA, compared to conventional multiple access schemes e.g., time-division multiple access (TDMA), is the ability of hosting multiple users in a single resource block e.g., a time-slot. By utilizing advanced signal processing techniques in the decoding process, such as a successive interference cancelation (SIC), it's possible to confront with the intra-user interference. Furthermore, time-sharing (TS) has been proposed for uplink NOMA, where different decoding orders of users are implemented for different portions of time. This enlarges the capacity region and provides increased fairness among users [3], [4].

Meanwhile, two important metrics for characterizing the performance of a wireless communication network are the sum and minimum throughput of the users. Usually these objectives are conflicted. In [5], the use of TDMA in WPN's was proposed, when the aim is to maximize the sum throughput and the circuit power consumption of these devices is non-negligible. However, this resource allocation leads to an unfair rate allocation among users, since the sum-data rate improvement favors users with better channel conditions, while users with weak channel conditions are almost prevented to access the resource block, as was observed in [6]. Furthermore, in [6], NOMA was proposed in WPNs with the goal to maximize the minimum throughput, which might lead though to a reduction of the sum throughput. As a matter of fact, it becomes evident that a trade-off between those two objectives i.e., sum throughput and minimum throughput, arises, since the maximization of one can lead to the decrease of the other. Meanwhile, NOMA and TDMA compete for their consolidation in WPNs.

In this direction, we are aiming to jointly optimize the sum and minimum throughput among users and try to identify when NOMA outperforms TDMA and vice-versa, for use in the uplink of WPNs. This problem belongs to the category of multi-objective optimization (MOO), while it can be transformed into a single-objective optimization problem, through the scalarization approach [7]. After solving the corresponding maximization problems, the Pareto front can be obtained, which is a widely accepted solution, when confronting with MOO problems [7]. The Pareto boundary describes the set of efficient potential operating points, while the network designer is responsible for selecting the point, which seems to be more appropriate for fulfilling the network requirements. Finally, simulation results exhibit the trade-off between the conflicted metrics, via the Pareto front, while the performance of both NOMA and TDMA is evaluated.

II. System Model

A wireless network is considered, consisting of N users and a single BS, while all nodes are equipped with a single antenna. The path loss factor from the BS to user n is denoted by L_n, while the channel coefficient is given by h_n, following a complex normal distribution, i.e., $h_n \sim \mathcal{CN}(0, 1)$. The communication is divided into time frames of unitary duration, with the channel state remaining constant during each time slot, while it can be perfectly estimated by the BS. We further consider the adoption of the harvest-then-transmit protocol, i.e., the amount of time $1-T$, $0 \leq T \leq 1$ is assigned to the BS to transfer wireless energy to all users, while the remaining time, T, is utilized for information transmission. More specifically, in the case of TDMA, each user transmits for a portion t_n of the transmission time T, while $T = \sum_{n=1}^{N} t_n$. On the other hand, in NOMA, all users simultaneously transmit information messages to the BS. For users' signals detection, the BS employs a joint processing technique, namely SIC according to the NOMA principle, where the already decoded messages are subtracted from the received signal. The available user transmission power is limited by the total harvested energy by each user during the first phase. Also, we consider channel reciprocity, so g_n is the same for both phases and it is given by $g_n = L_n |h_n|^2$. Without loss of generality we assume that $g_1 \geq g_2 \geq ... \geq g_N$. Finally, we assume that along with the transmit power, each device also consumes a constant power p_{c}, for the circuit operation.

A. Energy Harvesting Model

The energy harvesting model in WPNs, which allows users to harvest energy from the BS, could be considered as either linear or non-linear [8]. In both cases, the total harvested energy from the n-th user is a function of user's channel gain g_n and the transmission power of the BS, P_0. As a consequence, n-th user's total harvested power can be written as

$$\Phi_{\mathrm{EH},n} = f(g_n, P_0), \quad \forall n \in \mathcal{N}. \tag{1}$$

This is a general definition of the total harvested energy, while the appropriate model, which simulates the real conditions better, could be selected. In general, a non-linear energy harvesting model is more representative in practical conditions compared to a linear one [8].

B. TDMA Scheme - Information Transmission

Since in TDMA the users transmit information in different portions of time with duration t_n, the achievable throughput of user n, is given by

$$R_n^{\mathrm{TDMA}} = t_n \log_2(1 + \rho p_n g_n), \tag{2}$$

where p_n is the transmit power of user n and $\rho = 1/N_0$, with N_0 being the power spectral density of the additive white Gaussian noise (AWGN). The consumed power is constrained by

$$p_n + p_{\mathrm{c}} = \frac{E_n}{t_n} = \frac{\Phi_{\mathrm{EH},n}(1-T)}{t_n}, \quad \forall n \in \mathcal{N}. \tag{3}$$

By replacing p_n from (3) in (2), the achievable throughput of user n, can be expressed as

$$R_n^{\mathrm{TDMA}} = t_n \log_2\left(1 + \frac{1-T}{t_n}\rho g_n \Phi_{\mathrm{EH},n} - \rho p_c g_n\right). \tag{4}$$

Note that, by demanding $p_n \geq 0$, t_n is bounded by

$$t_n \leq \frac{\Phi_{\mathrm{EH},n}(1-T)}{p_c}, \quad \forall n \in \mathcal{N}. \tag{5}$$

The minimum throughput of the users can be written as

$$R_{\min}^{\mathrm{TDMA}} = \min_{n \in \mathcal{N}} (R_n^{\mathrm{TDMA}}), \tag{6}$$

while the sum throughput can be expressed as $R_{\mathrm{sum}}^{\mathrm{TDMA}} = \sum\limits_{n=1}^{N} R_n^{\mathrm{TDMA}}$.

C. NOMA Scheme - Information Transmission

The consumed power is constrained by

$$p_n + p_{\mathrm{c}} = \frac{E_n}{T} = \frac{\Phi_{\mathrm{EH},n}(1-T)}{T}, \quad \forall n \in \mathcal{N}. \tag{7}$$

Taking into account (7) and that $p_n \geq 0$, it holds that

$$0 < T \leq \frac{1}{1 + \frac{p_c}{\min(\Phi_{\mathrm{EH},n})}} = \frac{1}{1 + \frac{p_c}{\Phi_{\mathrm{EH},N}}} \triangleq B < 1. \tag{8}$$

Hereinafter, TS is considered. Thus, in contrast to fixed decoding order that corresponds to the corner points, any point in the capacity region of uplink NOMA can be achieved. The later is defined as the convex closed hull of all vectors $(R_1, R_2, ..., R_N)$ satisfying

$$\sum_{n \in \mathcal{M}_k} R_n \leq T \log_2\left(1 + \rho \sum_{n \in \mathcal{M}_k} p_n g_n\right), \forall k : \mathcal{M}_k \subseteq \mathcal{N}. \tag{9}$$

Thus, by using time-sharing, the minimum rate among users is given by [4]

$$R_{\min}^{\mathrm{NOMA}} = \min_{n \in \mathcal{N}} \left(\frac{T \log_2\left(1 + \rho \sum\limits_{i=n}^{N} p_i g_i\right)}{N + 1 - n} \right), \tag{10}$$

since among the subsets $\mathcal{M}_k, \forall k : \mathcal{M}_k \subseteq \mathcal{N}$ with the same cardinality, $R_{\min}^{\mathrm{NOMA}}$ is constrained by the one that consists of users with the lowest g_n. By using (7) and $p_i = \frac{\Phi_{\mathrm{EH},i}(1-T)}{T} - p_c$, (10) can be rewritten as

$$R_{\min}^{\mathrm{NOMA}} = \min_{n \in \mathcal{N}} \left(\frac{T \log_2\left(1 + \frac{1-T}{T}\rho \sum\limits_{i=n}^{N} g_i \Phi_{\mathrm{EH},i} - \rho p_{\mathrm{c}} \sum\limits_{i=n}^{N} g_i\right)}{N+1-n} \right). \tag{11}$$

Moreover, the system throughput is given by [4], [5]

$$R_{\mathrm{sum}}^{\mathrm{NOMA}} = T \log_2\left(1 + \frac{1-T}{T}\rho \sum_{i=1}^{N} g_i \Phi_{\mathrm{EH},i} - \rho p_{\mathrm{c}} \sum_{i=1}^{N} g_i\right). \tag{12}$$

III. Multi-Objective Optimization (MOO) and Pareto Front

A. Pareto Front

As mentioned above, our goal is to simultaneously optimize the sum and minimum throughput of the users. A widely accepted solution when dealing with MOO problems is to obtain the Pareto front. All Pareto optimal points have the property that is not possible to further increase one objective, without degrading any other. More specifically, the Pareto domination is defined as follows:

Definition 1: Pareto Domination: Let the vector-valued function $f(x) = [f_1(x), f_2(x), ..., f_m(x)]$, $f : X \to \mathbb{R}^m$. A feasible solution $u \in X$ is said to Pareto dominate another solution $v \in X$, in a maximization context, if and only if:
$f_i(u) \geq f_i(v), \quad \forall i \in \{1, 2, ..., m\}$
and $\exists j \in \{1, 2, ..., m\} : f_j(u) > f_j(v)$

A solution which is not dominated by any other solution is said to be Pareto optimal. MOO theory provides several approaches in order to convert a multi-objective problem into a single-objective, whose maximization results in a optimal Pareto point. A widely used approach is the scalarization method [7].

B. Scalarization approach

The scalarization approach combines the m objectives of a goal function f, into a scalar goal function. Consequently an MOO problem is converted to a single-objective optimization problem, as follows

$$\begin{aligned} \max_{\mathbf{x}} \quad & \sum_{i=1}^{m} w_i f_i(x), \\ \text{s.t.} \quad & \mathrm{C} : \forall \mathbf{x} \in \mathcal{X}, \quad \sum_{i=1}^{m} w_i = 1, \end{aligned} \tag{13}$$

where $w_1, w_2, ..., w_m$ are positive weights that specify the priority among the objectives. By tuning the weighting factors and by solving the maximization problem, any point in the Pareto boundary can be achieved.

IV. Joint Sum and Minimum Throughput Maximization

A. NOMA Scheme - Problem Formulation and Solution

The MOO problem can be formulated as follows, while we drop the superscript indices for simplicity

$$\begin{aligned} \max_{T, R_{\min}} \quad & (R_{\text{sum}}, R_{\min}) \\ \text{s.t} \quad & C_n : R_{\min} \leq \frac{T \log_2\left(1 + \frac{1-T}{T} \rho \sum_{i=n}^{N} g_i \Phi_{\text{EH},i} - \rho p_{\text{c}} \sum_{i=n}^{N} g_n\right)}{N-n+1}, \\ & \quad \forall n \in \mathcal{N}, \\ & C_{N+1} : 0 < T < B, \end{aligned} \tag{14}$$

where the constraints C_n in the above maximization problem occur from (11), while the constraint C_{N+1} is related with the transmission time bound in (8).

Following that, let

$$\tilde{R} = w R_{\text{sum}} + (1-w) R_{\min}, \quad \forall n \in \mathcal{N}, \quad 0 \leq w \leq 1, \tag{15}$$

where w denotes the weighting factor between the two objectives. From (15), $R_{\min}$ is given by

$$R_{\min} = \frac{\tilde{R} - w R_{\text{sum}}}{1-w}. \tag{16}$$

Since $R_{\min}$ is constrained by C_n in (14), (16) leads to

$$\begin{aligned} & \frac{\tilde{R} - w R_{\text{sum}}}{1-w} \leq \frac{T \log_2\left(1 + \frac{1-T}{T} \rho \sum_{i=n}^{N} g_i \Phi_{\text{EH},i} - \rho p_{\text{c}} \sum_{i=n}^{N} g_n\right)}{N-n+1}, \\ & \forall n \in \mathcal{N}, \end{aligned} \tag{17}$$

where R_{sum} is given by (12). Primarily, the optimization problem will be solved for a constant w, which corresponds to a specific point of the front, and subsequently one-dimension research will be carried out, in order to construct the complete Pareto boundary. Following that, we can re-formulate the maximization problem in (14) as follows

$$\begin{aligned} \max_{T, \tilde{R}} \quad & \tilde{R} \\ \text{s.t} \quad & C_n : \quad \tilde{R} \leq F_n(T), \quad \forall n \in \mathcal{N}, \\ & C_{N+1} : \quad 0 < T < B, \end{aligned} \tag{18}$$

where C_n occur by expanding the inequality in (17) and $F_n(T)$ is given by

$$\begin{aligned} F_n(T) = & \, wT \log_2\left(1 + \tfrac{1-T}{T} \rho \sum_{i=1}^{N} g_i \Phi_{\text{EH},i} - \rho p_{\text{c}} \sum_{i=1}^{N} g_i\right) \\ & + \left(\tfrac{1-w}{N-n+1}\right) T \log_2\left(1 + \tfrac{1-T}{T} \rho \sum_{i=n}^{N} g_i \Phi_{\text{EH},i} - \rho p_{\text{c}} \sum_{i=n}^{N} g_n\right) \\ & \forall n \in \mathcal{N}. \end{aligned} \tag{19}$$

In order to solve the maximization problem in (18), the proposed algorithm in [6] will be applied. It's easy to prove that the function F_n is concave with respect to T. As a consequence, there's a unique maximization point of the function F_n, which corresponds to the optimal time $T^* \in (0, B]$. According to [6], the optimal value of $\tilde{R}$, could be either the minimum of F_n maxima, or a possible intersection point between all the pairs of the function set, intersected below the min-max point, with different slopes.

In order to calculate the maximum of $F_n, \quad \forall n \in \mathcal{N}$, a numerical method, such as *bisection method*, should be applied, since a closed-form solution of the maximum is prevented. While, in order to search for all possible intersections between all the pairs of functions, the following set is constructed

$$\begin{aligned} G_{km}(T) &= F_k(T) - F_m(T) \\ &= (1-w)\left(\frac{T \log_2\left(1 + \frac{1-T}{T} a_k - b_k\right)}{N_k} - \right. \\ & \left. - \frac{T \log_2\left(1 + \frac{1-T}{T} a_m - b_m\right)}{N_m}\right), \end{aligned} \tag{20}$$

$\forall k, m \in \mathcal{N}, k \neq m$, where $T \in [0, B]$, $a_j = \rho \sum_{i=j}^{N} g_i \Phi_{\text{EH},i}$, $b_j = \rho p_{\text{c}} \sum_{i=j}^{N} g_i$ and $N_i = N + 1 - i$. By finding the roots of $G_{km}(T)$ it's possible to calculate all the intersections between all the pairs of functions of the set F_n. A numerical method for finding all the intersections is imperative. Taking these into account, we can construct the algorithm, as proposed in [6], in order to solve the maximization problem in (18). A brief description is presented in Algorithm 1.

In Step 2, the *bisection method* can be applied in order to search for the roots of $G_{km}(T)$. The search intervals of the *bisection method* have been specified in [6], where it has been proved that G_{km} has at most three roots, including zero point. With this algorithm it's possible to maximize $\tilde{R}$ and calculate the optimal transmission time $T^* \in (0, B]$, which achieves $\tilde{R}^*$. Note that the algorithm will be applied for a constant weighting factor w, while for every choice of w, a different point on the Pareto front is obtained. By sweeping w and by finding the optimal point $(T^*, \tilde{R}^*)$ in every step, we are able to characterize the whole Pareto boundary.

Algorithm 1 Solution of Maximization Problem in (18)

1: **Step 1:** Find all maxima of F_n, $\forall n \in \mathcal{N}$ and save the minimum of the maxima: $\tilde{R} = \min_{n \in \mathcal{N}}(\max F_n)$.
2: **Step 2:** Find all the intersections between all pairs of F_n, by finding the roots of (20).
3: **Step 3:** For all the intersections, check if the intersection point is smaller than $\tilde{R}$ and update $\tilde{R}$ with the value of the intersection point.

B. TDMA Scheme - Problem Formulation

Following the same pattern with the problem formulation of NOMA, the maximization problem in the case of TDMA can be similarly formulated, while it belongs to the category of convex-optimization problems. Thus, it can be solved with the aid of standard convex-optimization methods.

V. Performance Evaluation

A. Simulation Results

The users are uniformly distributed in a ring with inner radius $R_1 = 5$ m and outer radius $R_2 = 20$ m. In addition, the noise power spectral density has been set $N_o = -174$ dBm/Hz while the available bandwidth is 1 MHz at a carrier center frequency of 470MHz. The path loss model has been adopted similarly to [4]. Finally, the energy harvesting model is considered as non-linear, while the model's parameters have been set according to [8].

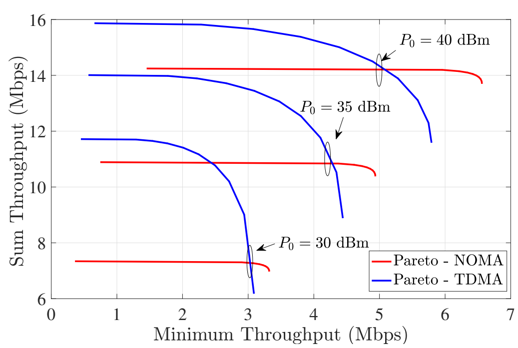

Fig. 1. Average Pareto Front for NOMA and TDMA with $p_c = 0.1\,\mathrm{mW}$.

In Fig. 1, the average Pareto front between sum throughput and minimum throughput, for various values of P_0, is illustrated, via Monte Carlo simulations. The circuit power consumption has been set $p_c = 0.1$mW, while $N = 2$ users have been considered to access the network. It is observed that NOMA outperforms TDMA when the goal is to offer fairness among users i.e., improve the minimum data rate, while TDMA performs better in terms of sum throughput. For the case of negligible circuit power consumption i.e., $p_c = 0$, NOMA totally dominates TDMA, since for equal sum throughput values, NOMA always presents higher minimum throughput, as it can be observed in Fig. 2. Note that in this case, both multiple access schemes, achieve the same maximum sum throughput, as was concluded in [4], while NOMA provides more fairness.

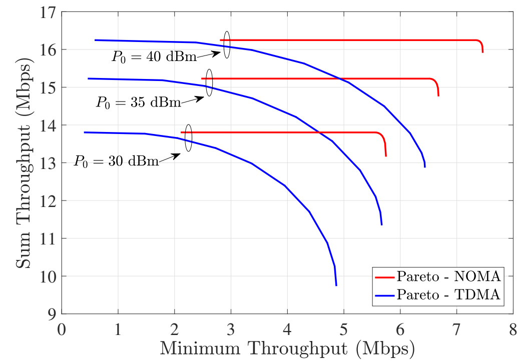

Fig. 2. Average Pareto Front for NOMA and TDMA with $p_c = 0$.

VI. Conclusion

In this work, we jointly optimized the system throughput and the minimum throughput among users in the uplink of a WPN. Furthermore, we investigated the use of NOMA and TDMA in WPNs and evaluated their performance in terms of both sum throughput improvement and fairness provision. To this end, by obtaining the Pareto front, the trade-off between the considered conflicted objectives is revealed.

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