# Outage Probability Estimation for a Multi-hop Terrestrial FSO Link Simplified to a Dual-hop Scheme

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***Abstract*— The last years, FSO technology has demonstrated an increasing scientific and commercial interest. However, due the signal's propagation through the atmosphere, the operation of FSO links depends strongly on the weather conditions and mostly by atmospheric turbulence effect. A common way for the maximization of the effective radius of FSO systems operating through the atmosphere is the employment of DF relays. However, by employing multiple relays and depending on the adopted fading model for turbulence, the complexity of the expressions used for the performance estimation of such systems can be significant increased. Motivated by the latter, in this work we minimize the mathematical complexity of these expressions by emulating the multi-hop system as dual-hop and by using an approximation for the turbulence-induced fading.**

***Keywords—FSO, multi-hop, DF relays, mixture gamma***

## I. INTRODUCTION

Free-Space-Optical (FSO) technology is the transfer of signals between two points by using optical radiation as the carrier signal through a channel without using any kind of wave-guidance and is essentially based on line-of-sight (LOS) [1]. Over the last few years, FSO has demonstrated increase in both commercial and research interest, because of many advantages they have compared to traditional Radio Frequency (RF) systems, [2]. Most important of them are the robustness to electromagnetic interference [3], the high bandwidth access, the operation in the unlicensed spectrum [3], the security [4] and the low operation and installation cost [2], [5]. However, one severe disadvantage of these systems is their strong dependence of the atmospheric conditions [4], [6]. The most significant performance mitigation factor among them, is turbulence which occurs because of inhomogeneities in atmospheric pressure and temperature [7], and causes rapid fluctuations in the received irradiance of a typical outdoor LOS optical link. Hence, the so-called scintillation effect, which arises by turbulence [8], causes to the atmospheric channel randomly time-varying characteristics, [1], [2].

Some well-known statistical models used for the description of the scintillation caused by turbulence effect in FSO systems are the Gamma-Gamma, the Málaga and the Negative Exponential, [6], [9], [10], and each one is appropriate for a specific range of turbulence conditions. Nevertheless, some of them may lead to mathematical expressions used for the performance estimation of high complexity. An alternative way for the description of turbulence-induced scintillation, is a recently presented model, namely Mixture Gamma (MG) distribution, which is appropriate for the description of the whole range of turbulence conditions by approximating various well-known statistical models, while at the same time has the notable advantage of being expressed through a Probability Density Function (PDF) with low mathematical complexity [2], [11].

A common way for extension of the effective radius and the counterbalancing of the coverage limitation due to turbulence-induced fading in FSO systems, is the employment of Decode and Forward (DF) relay nodes [2], [12], [13]. However, the employment of multiple relay nodes, can remarkably increase the mathematical complexity of the expressions used for the description and the total performance estimation of FSO systems operating over turbulent channels.

Motivated by the latter, and in order to minimize the mathematical complexity of these expressions, in this work, we emulate a multi-hop configuration as a dual-hop scheme [14], and we consider Málaga turbulent channels, approximated by MG model. More analytically, the remainder of this work is organized as follows: in section 2 we introduce the channel model composed by MG approximation of Málaga distribution and we analyze the dual-hop emulation of a multi-hop configuration. Next, in section 3 we proceed to the performance analysis of the DF relay assisted FSO system in terms of Outage Probability (OP), while the corresponding numerical results are presented in section 4. Finally, in section 5 we present some concluding remarks.

## II. CHANNEL MODEL

In this work, we consider the multi-hop DF relay-assisted FSO topology of Fig.1, where the source (S), which is assumed to be "node 0", emits the optical signal towards $R_{1...L}$ relay nodes and each of them emits the same signal towards the next relay node(s) or the receiver in the destination (D) which is assumed "node ($L$+1)", and we transform it to the simplified emulated dual-hop scheme of Fig. 2. Considering On Off Keying (OOK) modulation with Intensity Modulation and Direct Detection (IM/DD), the received signal at each of the hops is given as [2], [12], [13], [15]:

$$y_{x,y} = \eta I_{x,y} x_{x,y} + n_{x,y}, \tag{1}$$

where *x, y* represent the specific hop, i.e. from node *x* towards node *y*, $I_{x,y}$ stands for the normalized received irradiance in the corresponding receiver, $x_{x,y}$ is the corresponding modulated signal, $\eta$ is the effective photo-current conversion ratio, and $n_{x,y}$ stands for the Additive White Gaussian Noise (AWGN) with zero mean and variance $N_0/2$ [2], [12]. Turbulence-induced scintillation and path losses obligate in fluctuation the normalized received irradiance, which given as [12], [13], [15]:

$$I_{x,y} = I_{a,\mathrm{x,y}} I_{\mathrm{l,x,y}}, \tag{2}$$

where $I_{a,\mathrm{x,y}}$ and $I_{\mathrm{l,x,y}}$ represent the corresponding normalized received irradiance due to turbulence-induced scintillation and path losses respectively, in the specific hop. However, as the scope of this work is not the investigation of path losses, we consider $I_{\mathrm{l,x,y}} = 1$, [12].

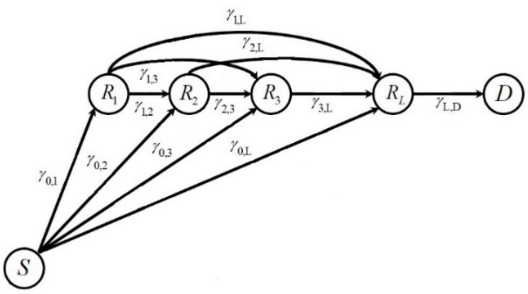


Fig. 1. Architecture of multi-hop DF relayed FSO system

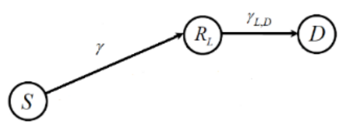


Fig. 2. Architecture of approximated DF relayed FSO system

Hence, the instantaneous Signal-to-Noise-Ratio (SNR) at the receiver of each hop can be written, as [2], [16]:

$$\gamma_{x,y} = \left(\frac{\eta I_{x,y}}{\sigma_n}\right)^2 \tag{3}$$

while the expected SNR is estimated as:

$$\overline{\gamma}_{x,y} = \left(\frac{\eta E[I_{x,y}]}{\sigma_n}\right)^2 \tag{4}$$

with, $E[I_{x,y}]$ being the corresponding expected normalized received irradiance value and considered normalized to unity.

*A. Approximation of Turbulence-induced Fading*

The PDF of MG distribution as function of irradiance $I_{x,y}$ at each hop is given as, [2], [11]:

$$f_{I_{\mathrm{x,y}}}(I_{\mathrm{x,y}}) = \sum_{i=1}^{N} w_i f_i(I_{\mathrm{x,y}}) = \\ = \sum_{i=1}^{N} a_i I_{\mathrm{x,y}}^{b_i - 1} e^{-\zeta_i I_{\mathrm{x,y}}}, I_{\mathrm{x,y}} > 0, \tag{5}$$

where $w_i = a_i \Gamma(b_i) \zeta_i^{-b_i}$ with $\sum_{i=1}^{N} w_i = 1$, $N$ is the number of the summation terms, and the larger the value of $N$ the smaller the divergence between a statistical distribution and it's approximation by MG model, [11]. Additionally, $f_i(x) = \frac{\zeta_i^{b_i} x^{b_i - 1} \exp(-\zeta_i x)}{\Gamma(b_i)}$ is the PDF of a Gamma distribution, [11], $a_i$, $b_i$, $\zeta_i$ are the parameters of the *i-th* Gamma component [2], and $\Gamma(.)$ stands for the Gamma function [11]. After a simple power transformation, the PDF expression for the SNR is written as:

$$f_{\gamma_{x,y}}(\gamma_{x,y}) = \frac{1}{2} \sum_{i=1}^{N} a_i \sqrt{\frac{\gamma_{x,y}^{b_i - 2}}{\overline{\gamma}^{b_i}}} \exp\left(-\zeta_i \sqrt{\frac{\gamma_{x,y}}{\overline{\gamma}_{x,y}}}\right). \tag{6}$$

The corresponding CDF is derived through the integral, [16]:

$$F_\gamma(\gamma) = \int_0^\gamma f_\gamma(\gamma) d\gamma. \tag{7}$$

By substituting equation (6) into (7) and using (Eq. 3.381.8) of Ref. [17] we get:

$$F_{\gamma_{x,y}}(\gamma_{x,y}) = \sum_{i=1}^{N} a_i \zeta_i^{-b_i} \gamma\left(b_i, \frac{\zeta_i \gamma_{x,y}^{1/2}}{\overline{\gamma}_{x,y}^{1/2}}\right). \tag{8}$$

Next, the PDF of Málaga distribution for each hop is given as, [10], [11], [18], [19] :

$$f_{I_{\mathrm{x,y}}}(I_{x,y}) = A_{x,y} \sum_{k'=1}^{\beta'_{x,y}} a'_{k',x,y} \tilde{I}_{x,y}^{\frac{\alpha'_{x,y} + k'}{2} - 1} \times \\ \times K_{\alpha'_{x,y} - k'}\left(2\sqrt{\frac{\alpha'_{x,y} \beta'_{x,y} \tilde{I}_{x,y}}{\gamma' \beta'_{x,y} + \Omega'}}\right), \tag{9}$$

where, $A_{x,y} = \frac{2 \alpha'^{\frac{\alpha'_{x,y}}{2}}_{x,y}}{\gamma'^{1 + \frac{\alpha'_{x,y}}{2}} \Gamma(\alpha'_{x,y})} \left(\frac{\gamma' \beta'_{x,y}}{\gamma' \beta'_{x,y} + \Omega'}\right)^{\beta'_{x,y} + \frac{\alpha'_{x,y}}{2}}$,

$a'_{k',\mathrm{x,y}} = \binom{\beta'_{x,y} - 1}{k' - 1} \frac{(\gamma' \beta'_{x,y} + \Omega')^{1 - \frac{k'}{2}}}{(k' - 1)!} \left(\frac{\Omega'}{\gamma'}\right)^{k' - 1} \left(\frac{\alpha'_{x,y}}{\beta'_{x,y}}\right)^{\frac{k'}{2}}$,

$\Omega' = \Omega + 2\rho b_0 + 2\sqrt{2\rho b_0 \Omega} \cos(\varphi_A - \varphi_B)$. In (9), $K_v(\cdot)$ is the *v*-order modified Bessel function of the second kind (equation (8.432.1) of Ref. [17]), and $\tilde{I}_{a,m} = I_{a,m} / E[I_{a,m}]$ with $E[I_{a,m}] = 1$. Furthermore, $\Omega$ stands for the average power of the LOS component, $\gamma' = 2b_0(1 - \rho)$, $2b_0$ is the average power of the total scatter components, $\alpha'_{x,y}$ is a positive parameter depending on the effective number of large-scale cells of the scattering process, and $\beta'_{x,y}$ is a natural number which represents the amount of turbulence. Moreover, $\rho$, with $0<\rho<1$, is the amount of scattering power coupled to the LOS component, and $\varphi_A$, $\varphi_B$ are the deterministic phases of the LOS and the coupled-to-LOS components.

In what follows, we consider the same parameters values in all links of the system, thus we omit the indices x, y. Expression (9) can be expressed as in Ref. [11] through the MG PDF of expression (5) with the following set of parameters:

$$a_i = \frac{\theta_i}{\sum_{j=1}^{N}\theta_j \Gamma(b_j)\zeta_j^{-b_j}},\ b_i = \alpha',\ \zeta_i = \frac{\alpha'\beta'}{(\gamma\beta'+\Omega')t_i},$$
$$\theta_i = \frac{A}{2} w_i \sum_{k'=1}^{\beta'} a'_{k'} \left(\frac{\alpha'\beta'}{\gamma\beta'+\Omega'}\right)^{\frac{\alpha'-k'}{2}} t_i^{k'-\alpha'-1}, \qquad (10)$$

where $w_i$ and $t_i$ stand for the weight factors and the abscissas, respectively [11].

### B. Approximation of multi-hop FSO system topology

By following the methodology of Ref. [14] in the MG PDF of equation (5), the new approximated PDF for the first hop of the system, i.e., from (S) towards node $R_L$ is obtained as:

$$f_{\bar{\Gamma}_r}(\gamma) = \sum_{p=1}^{r} \pi_{r,p} \sum_{i=1}^{N} a_i \sqrt{\frac{\gamma^{b_i-2}}{4\bar{\Gamma}^{b_i}}} \exp\left(-\zeta_i \sqrt{\frac{\gamma}{\bar{\Gamma}_{r,p}}}\right) d\gamma \qquad (11)$$

with $\left.\frac{1}{\bar{\Gamma}_{r,p}}\right|_{p=1}^{r-1} = \frac{1}{\bar{\Gamma}_{r-1,p}} + \frac{1}{\bar{\gamma}_{r-1,r}}$ , $\left.\pi_{r,p}\right|_{p=1}^{r-1} = \frac{\pi_{r-1,p}\bar{\Gamma}_{r,p}}{\bar{\Gamma}_{r,p} - \bar{\Gamma}_{r,r}}$,

$\bar{\Gamma}_{r,r} = \bar{\gamma}_{0,r}$ , $\pi_{r,r} = \sum_{p=1}^{r-1} \frac{\pi_{r-1,p}\bar{\Gamma}_{r,r}}{\bar{\Gamma}_{r,r} - \bar{\Gamma}_{r,p}}$, $\pi_{1,p} = 1$, with $r$ being the number of DF relay nodes, $\gamma$ stands for the instantaneous SNR, and $\bar{\Gamma}_{r,p}$ represents the approximated expected SNR .

The Cumulative Distribution Function (CDF) is obtained after the introduction of (11) into (7) and by using (equation (3.381.8) of Ref. [17]) as:

$$F_{\bar{\Gamma}_r}(\gamma) = \sum_{p=1}^{r}\sum_{i=1}^{N} \frac{\pi_{r,p} a_i}{\zeta_i^{b_i}} \gamma\left(b_i, \zeta_i \sqrt{\frac{\gamma}{\bar{\Gamma}_{r,p}}}\right) \qquad (12)$$

where $\gamma(\cdot,\cdot)$ represents for the lower incomplete Gamma function, [Eq. 8.310.1 of Ref. [17]].

## III. Outage Probability Estimation

In this section we investigate the performance of the whole system, by using the Probability of Outage (OP) metrics. OP represents the probability that the instantaneous SNR falls below a critical threshold, $\gamma_{th}$, which corresponds to the sensitivity limit of each receiver [2], [20]. Thus, the OP is estimated as, [12], [20]:

$$P_{out,\mathrm{r}} = \Pr(\gamma \le \gamma_{th}) = F_{\bar{\Gamma}_r}(\gamma_{th}) \qquad (13)$$

By using the above derived CDF i.e., expression (12), into expression (13) above, the OP of the first hop i.e. from (S) towards $R_L$ is obtained as:

$$P_{out,\mathrm{r}} = \sum_{p=1}^{r}\sum_{i=1}^{N} \frac{\pi_{r,p} a_i}{\zeta_i^{b_i}} \gamma\left(b_i, \zeta_i \sqrt{\frac{\gamma_{th}}{\bar{\Gamma}_{r,p}}}\right) \qquad (14)$$

Next, the OP of the second link, i.e., from $R_L$ towards the destination (D) can be given by the introduction of (8) onto (13) as:

$$P_{out,D} = \sum_{i=1}^{N} a_i \zeta_i^{-b_i} \gamma\left(b_i, \zeta_i \sqrt{\frac{\gamma_{th}}{\bar{\gamma}}}\right). \qquad (15)$$

The total OP of the whole system can be derived as [12], [13], [15]:

$$P_{out} = 1 - [1 - P_{out,\mathrm{r}}][1 - P_{out,\mathrm{D}}] \qquad (16)$$

and after the substitution of (14) and (15) into (16) we get:

$$P_{out} = 1 - \left[1 - \sum_{p=1}^{r}\sum_{i=1}^{N} \frac{\pi_{r,p} a_i}{\zeta_i^{b_i}} \gamma\left(b_i, \zeta_i \sqrt{\frac{\gamma_{th}}{\bar{\Gamma}_{r,p}}}\right)\right] \times \left[1 - \sum_{i=1}^{N} \frac{a_i}{\zeta_i^{b_i}} \gamma\left(b_i, \zeta_i \sqrt{\frac{\gamma_{th}}{\bar{\gamma}}}\right)\right] \qquad (17)$$

## IV. Numerical Results

In this section, we demonstrate some indicative numerical results, by using the parameters set of (10) into the above derived expression (17), and we consider the scenario, with three relay nodes. It should be mentioned here, that as in Ref. [14], for sake of simplicity and due to FSO system restrictions we ignore the link from $R_1$ towards $R_3$ . Furthermore, by considering $\alpha' = 2$ and $\beta' = 5$ ,or $\alpha' = 1$ and $\beta' = 2$ along with $N = 10$, the accuracy between the real Málaga distribution, and it's approximation by MG model is ensured for both these turbulence intensities [11]. Hence finally, for our configuration with three relays we have that $\bar{\gamma}_{0,1} = \bar{\gamma}_{1,2} = \bar{\gamma}_{2,3} = \bar{\gamma}_{3,4} = \bar{\gamma}_{L,D} = \bar{\gamma}$, $\bar{\gamma}_{0,2} = \bar{\gamma}$, and $\bar{\gamma}_{0,3} = \bar{\gamma}$.

Fig.3 depicts the final OP versus the normalized outage threshold $\bar{\gamma}/\gamma_{th}$ of a three DF relay assisted FSO system, emulated as the dual-hop of Fig. 2, over Málaga turbulent channels approximated by MG model. Additionally, we provide the corresponding numerical results for the real three relayed system without using the dual-hop emulation, in order to test the accuracy of this method. In this configuration, the total OP of the system, by appropriate using [12], can be written as:

$$P_{tot} = 1 - \left[ (1 - P_{out,D})(1 - P_{out,D}(1 - (1 - P_{out,D})(1 - P_1))) \right] \quad (18)$$

where $P_1 = \left[ 1 - (1 - P_{out,D})^2 \right] P_{out,D}$. As shown in Fig. 3 the numerical results with or without dual-hop emulation are in almost complete agreement.

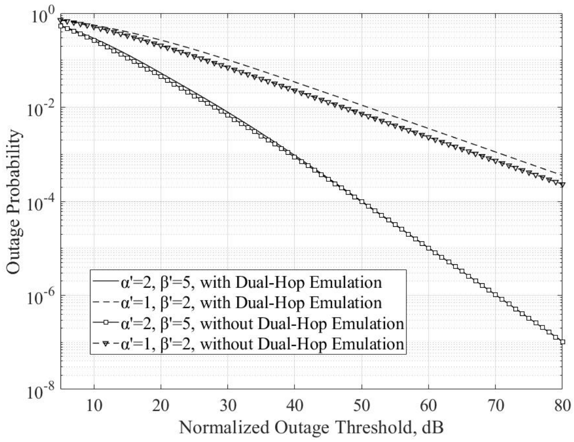


Fig. 3. OP of a three relayed system with and without using the simplification procedure.

## V. CONCLUSIONS

In this work, an effective way for the minimization of the mathematical complexity of the expressions used for the performance estimation of DF relay assisted FSO systems over Málaga modeled turbulence channels presented. The corresponding numerical results were graphically depicted and demonstrated almost complete agreement between the real multi-hop and the dual-hop emulated system. The work can readily be extended to consider the employment of other turbulence approximations via MG model, and the evaluation of different performance metrics.


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## REFERENCES


[1] Z. Ghassemlooy and W. O. Popoola, "Terrestrial Free-Space Optical Communications," in *Mobile and Wireless Communications Network Layer and Circuit Level Design*, InTech, 2010.

[2] N. A. Androutsos, H. E. Nistazakis, H. Khalid, S. S. Muhammad, and G. S. Tombras, "Serial DF Relayed FSO Links over Mixture Gamma Turbulence Channels and Nonzero Boresight Spatial Jitter," *Computation*, vol. 7, no. 3, p. 34, Jul. 2019.

[3] M. Uysal, C. Capsoni, Z. Ghassemlooy, A. Boucouvalas, and E. Udvary, Eds., *Optical Wireless Communications*. Cham: Springer International Publishing, 2016.

[4] G. T. Djordjevic, M. I. Petkovic, M. Spasic, and D. S. Antic, "Outage capacity of FSO link with pointing errors and link blockage," *Opt. Express*, vol. 24, no. 1, p. 219, Jan. 2016.

[5] Z. Ghassemlooy, S. Arnon, M. Uysal, Z. Xu, and J. Cheng, "Emerging Optical Wireless Communications-Advances and Challenges," *IEEE J. Sel. Areas Commun.*, vol. 33, no. 9, pp. 1738–1749, Sep. 2015.

[6] M. P. Ninos, H. E. Nistazakis, and G. S. Tombras, "On the BER performance of FSO links with multiple receivers and spatial jitter over gamma-gamma or exponential turbulence channels," *Optik (Stuttg).*, vol. 138, pp. 269–279, 2017.

[7] G. T. Djordjevic and M. I. Petkovic, "Average BER performance of FSO SIM-QAM systems in the presence of atmospheric turbulence and pointing errors," *J. Mod. Opt.*, vol. 63, no. 8, pp. 715–723, 2016.

[8] M. A. Khalighi and M. Uysal, "Survey on Free Space Optical Communication: A Communication Theory Perspective," *IEEE Commun. Surv. Tutorials*, vol. 16, no. 4, pp. 2231–2258, 2014.

[9] G. K. Varotsos, H. E. Nistazakis, C. K. Volos, and G. S. Tombras, "FSO links with diversity pointing errors and temporal broadening of the pulses over weak to strong atmospheric turbulence channels," *Optik (Stuttg).*, vol. 127, no. 6, pp. 3402–3409, 2016.

[10] A. Jurado-Navas, J. Maria, J. Francisco, and A. Puerta-Notario, "A Unifying Statistical Model for Atmospheric Optical Scintillation," in *Numerical Simulations of Physical and Engineering Processes*, InTech, 2011.

[11] H. G. Sandalidis, N. D. Chatzidiamantis, and G. K. Karagiannidis, "A Tractable Model for Turbulence- and Misalignment-Induced Fading in Optical Wireless Systems," *IEEE Commun. Lett.*, vol. 20, no. 9, pp. 1904–1907, 2016.

[12] G. Varotsos, H. E. Nistazakis, A. N. Stassinakis, C. K. Volos, V. Christofilakis, and G. S. Tombras, "Mixed Topology of DF Relayed Terrestrial Optical Wireless Links with Generalized Pointing Errors over Turbulence Channels," *Technologies*, vol. 6, no. 4, p. 121, 2018.

[13] K. Prabu and D. S. Kumar, "Outage Analysis of Relay - Assisted BPSK - SIM Based FSO Systems over Strong Atmospheric Turbulence with Pointing Errors," vol. 3, no. 5, pp. 3–6, 2014.

[14] S. Lim and K. Ko, "Approximation of multi-hop relay to dual-hop relay and its error performance analysis," *IEEE Commun. Lett.*, vol. 21, no. 2, pp. 342–345, 2017.

[15] M. Feng, J. Wang, M. Sheng, L. Cao, and X. X. M. Chen, "Outage Performance for Parallel Relay-Assisted Free-Space Optical Communications in Strong Turbulence with Pointing Errors," pp. 7–11, 2011.

[16] H. G. Sandalidis, N. D. Chatzidiamantis, G. D. Ntouni, and G. K. Karagiannidis, "Performance of free-space optical communications over a mixture composite irradiance channel," *Electron. Lett.*, vol. 53, no. 4, pp. 260–262, 2017.

[17] I. S. Gradshteyn and I. M. Ryzhik, "Table of Integrals, Series, and Products (Academic Press)", 7th ed., New York, Elsevier, 2007.

[18] K. Prabu and S. Thakkar, "Analysis of FSO link with time diversity over M-distribution channel model with pointing errors and GVD effects," *Opt. Commun.*, vol. 421, no. November 2017, pp. 115–124, 2018.

[19] K. A. Balaji and K. Prabu, "Performance evaluation of FSO system using wavelength and time diversity over malaga turbulence channel with pointing errors," *Opt. Commun.*, vol. 410, no. July 2017, pp. 643–651, 2018.

[20] A. N. Stassinakis, H. E. Nistazakis, and G. S. Tombras, "Comparative performance study of one or multiple receivers schemes for FSO links over gamma-gamma turbulence channels," *J. Mod. Opt.*, vol. 59, no. 11, pp. 1023–1031, 2012.