A Chaotic Circuit with Bi-Color LED as a Nonlinear Element

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***Abstract*— In this work, a simple 3D chaotic circuit is experimentally studied. The proposed circuit is described by a 3-D dynamical system with two nonlinear terms, which are the same hyperbolic sine functions implemented with simple Bi-color LEDs. The proposed chaotic circuit presents interesting chaos related phenomena, like period doubling route to chaos, coexisting attractors and antimonotonicity.**

Keywords— ***Bi-color LED; Nonlinear circuit; chaos; antimonotonicity***

I. INTRODUCTION

Over the last three decades, chaos theory was intensively studied by researchers and analog circuit design engineers. Chaotic circuits can be easily constructed if the designer knows the set of describing differential equations. Universal method how to do this, is known as the analog computer concept and is thoroughly described in [1]. However, formation of self-excited strange attractors was reported within dynamics of standard, naturally non-chaotic structures of analog radio-frequency functional blocks. Robust chaos was unfolded via well-known numerical algorithms, such as the algorithm for calculating the Lyapunov Exponents (LEs), which can be applied on the output data or accidentally during experimental measurement. Dense chaotic attractors were observed and consequently numerically confirmed in the case of Colpitts [2], Hartley [3], Wien-bridge [4] harmonic oscillator, Chua circuit [5], Van der Pol oscillator [6], phase-locked loops [7], dc-dc converters [8], etc.

Many research teams are working today in the direction of enrichment of existing systems' complexity by modifying their nonlinear terms or adding more nonlinear terms. In this way systems with more complex dynamical behavior are produced, with high applicability to problems like secure communication systems and cryptography [9,10].

In literature there are three basic methods of chaotic systems' optimization regarding their complexity.

- As a first approach, one of the system's nonlinear terms is replaced by a higher order one, for example by changing the product term to a exponential or logarithmic function [11,12].
- In the second method a nonlinear term can be slightly adjusted, without changing its order [13,14].
- Finally, in the third method more nonlinear terms in systems, especially in simple systems which are described with few terms, are added. In this way the number of nonlinear terms and as a consequence the system's complexity is increased [15].

Furthermore, in the last few years chaotic circuits with a hyperbolic sine term as a nonlinear term, have been reported in the literature [16-20]. This nonlinear term can be easily constructed by using two antiparallel diodes. Interesting phenomena related to chaos theory, such as period doubling route to chaos, coexisting attractors and intermittency, have been observed, due to the nature of the *i-v* characteristic of the aforementioned nonlinear term.

In this work, the third method for enhancing the complexity of a known chaotic system has been adopted. The designed circuit, which has been constructed in order to emulate the proposed system, has only two nonlinear terms, which are the same hyperbolic sine functions implemented with simple Bi-color LEDs, instead of antiparallel diodes. This approach is very promising as it is easy to use this kind of nonlinear element in circuit's design and also very useful for chaos related applications, such as cryptography and secure communications.

The rest of the paper is organized as follows. In Section II the dynamical system and its circuit design are presented. The dynamical characteristics of the system, such as dissipation and symmetry as well as system's equilibrium points are investigated in Section III. Furthermore, the experimental results of circuit's operation confirming interesting phenomena related to chaos theory are presented in Section IV. Finally, Section V of this paper includes the conclusions of this work and some thoughts for future work.

II. The Proposed Chaotic Circuit

In 2017 a simple 3D chaotic system, belonging to the family of jerk systems has been reported [16]. This system was one of the first chaotic systems with a hyperbolic sine function as a nonlinear term. By adding one more hyperbolic sine function $\left(c\sinh(z)\right)$ in the second equation of the aforementioned system the following system is produced.

$$\begin{cases} \dfrac{dx}{dt} = -y \\ \dfrac{dy}{dt} = -z - c\sinh(z) \\ \dfrac{dz}{dt} = -x + a\sinh(y) - bz \end{cases} \tag{1}$$

The circuit that has been designed in order to emulate system (1) consists of three capacitors, ten resistors and five operational amplifiers (TL084CN), from which three of them (U_1 − U_3) are configured as integrators. Its nonlinear elements are two Bi-color LEDs. The current, through each of the Bi-color LEDs, is described by the equation:

$$I = 2I_S \sinh\left(\frac{\upsilon}{nV_T}\right) \tag{2}$$

which is produced by applying Kirchhoff's current law and the known Shockley diode equation for the two antiparallel LEDs that consists the Bi-color LED. In Eq. (2), n is a diode ideality factor, I_S is the reverse bias saturation current, υ is a voltage over the LEDs and V_T is a thermal voltage.

Therefore, the circuit that has been designed in order to emulate system (1) is presented in Fig. 1. Also, in Fig. 2 the experimental realization of the proposed circuit of Fig. 1, with two Bi-color LEDs is presented. The mathematical model given by the system (1) is obtained by applying the Kirchhoff's laws into the circuit of Fig. 1, in which $x = \dfrac{\upsilon_{C1}}{nV_T}$, $y = \dfrac{\upsilon_{C2}}{nV_T}$, $z = \dfrac{\upsilon_{C3}}{nV_T}$, $\tau = \dfrac{t}{RC}$, $a = \dfrac{2R_a I_S}{nV_T}$, $b = \dfrac{R}{R_b}$ and $c = \dfrac{2RI_S}{nV_T}$.

By using Bi-color LEDs the value of a and c are fixed as 4×10^{-4} and 3.846×10^{-4} respectively, according to the Bi-color LEDs specifications (I_S = 1nA, V_T = 26mV and n = 2). Note that the system's dynamical behavior can be adjusted by changing parameter b, which does not affect the Bi-color LED's Eq. (2). The rest of the circuit's elements have the following values: $C_1 = C_2 = C_3 = 10$ nF, $R = 10$ kΩ, $R_1 = 1$ MΩ, $R_a = 10.4$ kΩ and R_b: variable resistor, while the power supply is ±15 V.

Especially, comparing the maximum value of the *MLE* of this system ($MLE_{max} = 0.3904$), which is produced for $b = 0.625$, regarding the respective value of the system with one hyperbolic sinusoidal term of Ref. [15], which was 0.2250, a significant increase has been achieved. So, by

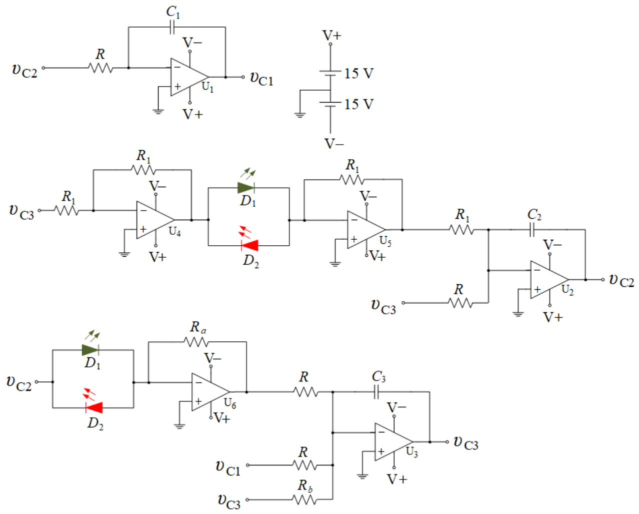

Fig. 1. Schematic of the proposed circuit of system (1).

adopting the third method of chaotic systems' optimization the complexity has been increased. In Fig. 5 the good agreement between the phase portrait for b = 0.625 is proved from the comparison of system's (1) numerical simulation with circuit's of Fig. 2 experimental observation.

III. Theoretical Analysis of the System

The divergence of system (1) is defined as:

$$\nabla V = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} = -b \tag{3}$$

where V indicates the phase volume. Since (3) is negative for all x, y, z and all $b > 0$, the system is bounded. Thus, system (1) is dissipative and converges in the index

$$\frac{dV}{dt} = e^{-bt} \tag{4}$$

The interpretation of this index is that each volume, containing the trajectories of (1), will reduce to zero as time approaches infinity, at an exponential rate of $V_0 e^{-bt}$. Thus, each trajectory of (1) is ultimately confined to a particular subset having zero volume, and its asymptotic motion of (1) is arranged to an attractor.

Moreover, the system is invariant under coordinate transformation $(x, y, z) \rightarrow (-x, -y, -z)$. So, if (x, y, z) is a solution of (1) for a choice of parameters, then $(-x, -y, -z)$ is also a solution for the same parameters. This means that when projected onto the (x, y, z) space, attractors are symmetrically inverted with respect to the origin. This symmetry could justify the phenomenon of several coexisting attractors in the state space.

Overall, it is seen that the system (1) is dissipative with coexisting attractors, two sought out features for novel chaotic systems.

By calculating system's (1) equilibria, we can find that system has three equilibria, for $a = 4\times10^{-4}$ and $c = 3.846\times10^{-4}$, at $E_1(-10.946b, 0, -10.946)$, $E_2(0, 0, 0)$ and $E_3(10.946b, 0, 10.946)$.

IV. Circuit's Dynamical Analysis

In this section, the dynamical analysis of the system (1) with respect to the bifurcation parameter b is performed. The other parameter values $a = 4\times10^{-4}$ and $c = 3.846 \times 10^{-4}$,

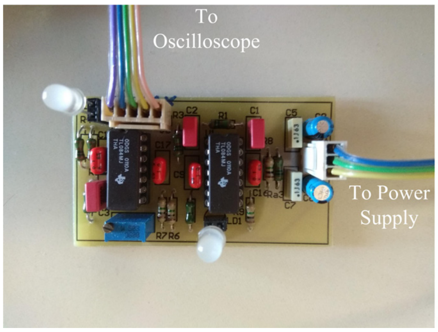

Fig. 2. The experimental realization of the proposed circuit of Fig.1.

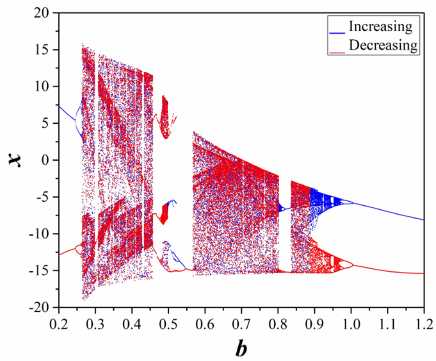

Fig. 3. Bifurcation diagrams of system (1), when parameter b is increased (red) and decreased (blue).

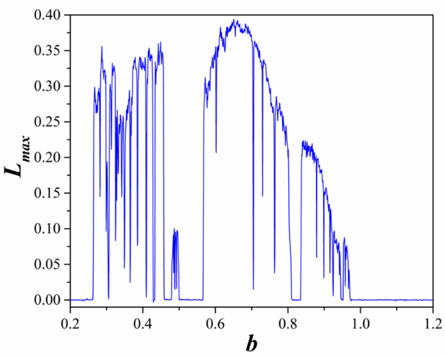

Fig. 4. Maximal Lyapunov exponents of system (1), when varying b.

with initial conditions $(x_0, y_0, z_0) = (0, 0.1, 0)$. For this investigation, system (1) is solved by implementing the fourth-order the Runge-Kutta algorithm, with fixed time step $\Delta t = 0.001$. With the help of the bifurcation diagram of Fig. 3, either the parameter b increasing or decreasing, and the Maximal Lyapunov Exponent (MLE) spectrum (Fig. 4), the circuit's dynamics can be directly analyzed with respect to changes in the parameter b.

V. Experimental Results

In this section, the experimental phase portraits of υ_{C2} versus υ_{C1}, by using a digital oscilloscope, for various values of the resistor R_b are presented, in order to confirm the simulated behavior of the bifurcation diagram of Fig. 3. In more detail, from the bifurcation diagram of Fig. 3, the period doubling route to chaos as the value of value of resistor R_b increases, is presented. This phenomenon is verified by the experimental phase portraits of Figs. 6b-6e.

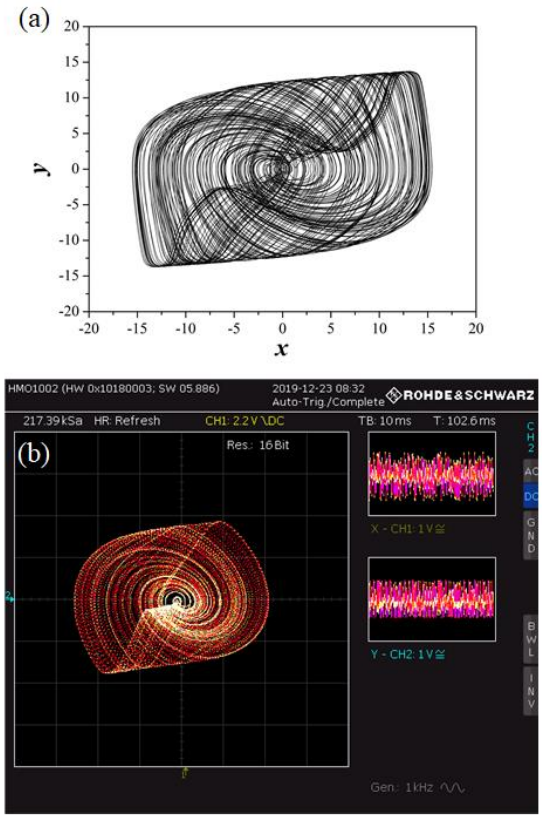

Fig. 5. Chaotic phase portraits of y versus x, for b = 0.625, produced (a) from system's (1) simulation and (b) from circuit's of Fig. 2 experimental observation.

Also, another interesting phenomenon can be observed in the bifurcation diagram of Fig. 3. For very low or very high values of parameter b, a coexistence behavior of system (1) is illustrated, which is experimentally observed from the phase portraits of Figs. 6a and 6b, for R_b = 50 kΩ (b = 0.2), in which two coexisting periodic attractors has been produced by turning on and off the power supply. In this way, the circuit has different initial conditions and the attractor is placed in different area in the phase plane.

Furthermore, looking at Fig. 3, the phenomenon of antimonotonicity is observed. Antimonotonicity, was first introduced by Dawson *et al.* [21], and refers to the phenomenon where the system enters into chaotic behavior via a period doubling route (p-1 → p-2 → ... → chaos) and exits from chaotic behavior by following a reverse, period halving route (chaos → ... → p-2 → p-1). This behavior is signified by a chaotic bubble shape in the bifurcation diagram. This phenomenon is confirmed as the circuit enters to chaos by following a period doubling sequence, as previously mentioned, and exits from chaos by following a reverse period doubling sequence (Figs. 6h-6j).

Finally, the periodic window in the bifurcation diagram around the value of b = 0.55 has been experimentally captured in Fig. 6g, which is intermediate between two chaotic regions, as is observed in Figs. 6f and 6h.

VI. Conclusion

In this work, a simple autonomous chaotic circuit with only two nonlinear terms, which are hyperbolic sinusoidal functions, was presented. These nonlinear terms can be

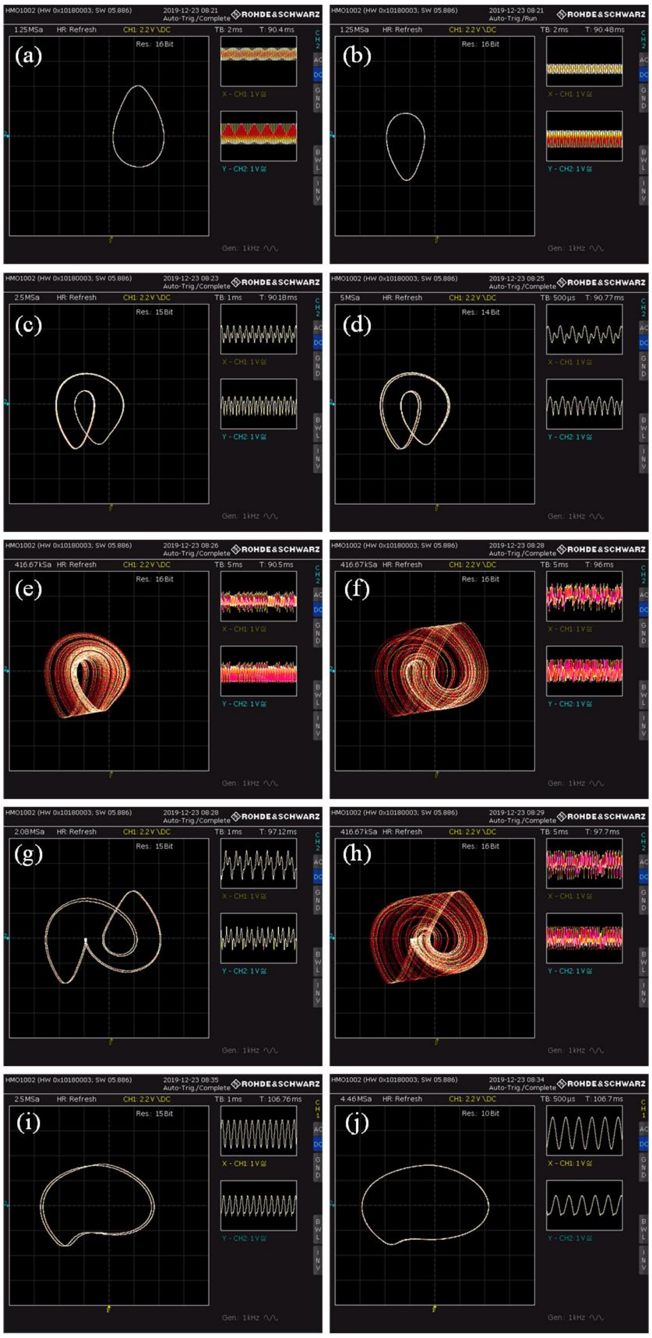

Fig. 6. Experimental phase portraits of υ_{C2} versus υ_{C1}, for (a) R_b = 50 kΩ (period-1), (b) R_b = 50 kΩ (coexisting period-1), (c) R_b = 40 kΩ (period-2), (d) R_b = 38.16 kΩ (period-4), (e) R_b = 38 kΩ (chaos), (f) R_b = 25 kΩ (chaos), (g) R_b = 19.05 kΩ (period-2), (h) R_b = 14.3 kΩ (chaos), (i) R_b = 10.1 kΩ (period-2) amd (j) R_b = 9.09 kΩ (period-1).

easily implemented with bicolor LEDs as it was discussed. The 3D system, which described the nonlinear autonomous circuit, presented a plethora of phenomena related to chaos, like antimonotonicity, period doubling route to chaos and coexisting attractors. Finally, the proposed circuit, was very simple in form yet complex, and can thus work as a basis for future works related with chaotic synchronization and secure communication systems.

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