# A Modified Tantalum Oxide Memristor Model for Neural Networks with Memristor-Based Synapses

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***Abstract* — This paper presents an improved modification of tantalum oxide memristor model and its application in neural networks. The proposed model is based on the standard Hewlett Packard tantalum oxide model with three improvements – application of a modified Biolek window function, optimization of its performance using simplified current-voltage relationship and by replacements of step model's components by continuous differentiable functions. The optimal values of the tuning model's coefficients are derived by comparison with experimental data and parameter estimation algorithm. A PSpice library memristor model is created in accordance to its mathematical model. The considered memristor model is applied in a simple neural network for function fitting with memristor-based synapses. A comparison with several existing tantalum oxide memristor models is made and the main advantages of the proposed model are established – higher performance, improved tuning capability and operation for hard-switching mode.**

***Keywords— tantalum oxide memristor, improved window function, PSpice library model, neural network.***

## I. INTRODUCTION

The memristors based on transition metal oxides have stable properties and many applications, as in nonvolatile computer memories, neuromorphic circuits, logical schemes, analog and digital configurable devices [1], [2], [3]. Among the resistance switching materials so far analyzed, as non-stochiometric titanium dioxide, hafnium oxide and silicon oxide [1], [4], the tantalum oxide partially doped by oxygen vacancies represents good resistance switching properties, mainly endurance, low voltage and current, high switching speed, low power consumption, long retention time and a sound compatibility with the CMOS integrated circuits technology [5], [6], [7]. The tantalum oxide based memristors contain a conducting channel and a partially depleted by oxygen vacancies region [8], [9]. The conductance and the respective memristor state could be changed by externally applied voltage [7]. To accurately express the tantalum oxide memristors behavior in electronic circuits a precise model is needed [8]. Several attempts for application of titanium dioxide memristor models for approximate modeling of tantalum oxide resistance switching elements exist in the literature [7], [8]. However the structure and principal of operation of TaO and $TiO_2$ memristors are different [7]. Due to this reason several special TaO memristor models are generated so far [8], [9], [10]. The Hewlett Packard standard model [8] uses step functions in the state differential equation and a modulus function in the current-voltage relationship, which unfortunately are continuous but not differentiable [9], [10]. This is a disadvantage of the model if it is used for PSpice library model creation due to occurrence of convergence problems [8], [9]. An improvement of this model is proposed in [9] by Ascoli, Tetzlaff and Chua. This modified model uses continuous and differentiable functions in the model equations instead of the described above non-differentiable expressions in [8]. Although the modified model [9], [10] is appropriate for PSpice incorporation it is a complex and time consuming one and requires many elementary calculations due to the large number of exponents and hyperbolic sine function in the state equation [9]. No window function has been used and sometimes the state variable is not limited in the interval [0, 1] which is another disadvantage of the above mentioned models. Another model proposed in [11], [12] is applicable for tantalum oxide memristor modeling. Unfortunately it uses non-differentiable step functions and due to this it has convergence problems [11]. The motivation for the present research is the partial absence of a simple and accurate TaO memristor model. The purpose of this investigation is to propose a precise, tunable and simplified model with window function [13], [14] for TaO memristors, applicable for PSpice library model generation. For adjustment of the offered memristor model experimental data [6] and algorithms for parameters estimation [15], [16] are applied. The criterion used for optimization the model is the minimum of the mean square error between the experimental and the simulated current-voltage characteristics [16]. A PSpice [17] library memristor model is created using the proposed modified mathematical model. The model is applied and successfully tested in a simple memristor neural network for function fitting [18], [19], [20].

The rest of the paper is organized as follows. In Section 2 a brief description of the structure, principle of operation and of the basic models of TaO memristors is made. The adjustment of the proposed memristor model made by the use of experimental current-voltage relationships, a methodology for optimal coefficients determination and an algorithm for parameters estimation in MATLAB - Simulink is described in Section 3. The corresponding PSpice memristor model is presented in Section 4. The operation of the TaO memristor model in a simple neural network with memristor-based synapses is described in Section 5. The concluding remarks are given in Section 6.

## II. A DESCRIPTION OF THE BASIC TANTALUM OXIDE MEMRISTOR MODELS AND THE PRESENT MODIFICATION

A schematic of the tantalum oxide memristor structure is presented in Fig. 1. It contains two metallic electrodes – anode and cathode, respectively [7], [8]. The element has a square intersection. Several parallel channels are existing in the memristor structure [7]. The external channel is formed by pure isolating $Ta_2O_5$. The internal channel is made by a solid solution of oxygen atoms in tantalum material – Ta(O) with high conductance. Between these two channels an intermediate partial channel of non-stochiometric tantalum oxide doped by oxygen vacancies exists [7], [8].

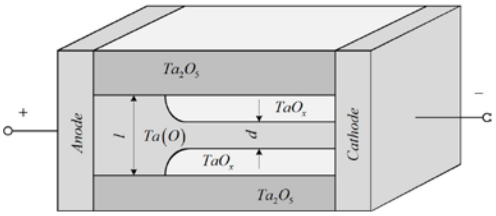

Fig. 1 Structure of tantalum oxide memristor

The memristor state could be changed by the use of external voltage signal. The memristor state variable *x* [8] is defined as a ratio between the area of the conducting channel $S_1$ and those of the whole memristor intersection $S_2$:

$$x = \frac{S_1}{S_2} = \frac{d^2}{l^2} \tag{1}$$

### A. The Standard Hewlett Packard Memristor Model [8]

The conductance of the internal channel when its area is maximal is denoted by *G* and it has a comparatively high value [7]. The conductance of the doped channel could be expressed by the nonlinear Frenkel–Poole relation [5], [7], [8]. The standard Hewlett Packard model for tantalum oxide memristors [8] includes the memristor equivalent conductance $G_{eq}$ as a parallel connection of the described channels including the state variable *x*:

$$i = v\,G_{eq} = v\left[G\,x + a\exp\left(b\sqrt{|v|}\right)(1-x)\right] \tag{2}$$

where *a* and *b* are fitting parameters. The state differential equation of the memristor element is [8], [9]:

$$\begin{aligned}\frac{dx}{dt} = {} & B\sinh\left(\frac{v}{\sigma_{ON}}\right)\exp\left(-\frac{x^2}{{x_{ON}}^2}\right)\exp\left(\frac{v\,i}{\sigma_P}\right)stp(v) + \\ & + A\sinh\left(\frac{v}{\sigma_{OFF}}\right)\exp\left(-\frac{{x_{OFF}}^2}{x^2}\right)\exp\left(\frac{1}{1+\beta\,i\,v}\right)stp(-v)\end{aligned} \tag{3}$$

where $B$, $A$, $\sigma_{OFF}$, $\sigma_{ON}$, $\sigma_P$, $\beta$, $x_{ON}$, and $x_{OFF}$ are fitting parameters [8], [10]. The step function *stp* is [9]:

$$\begin{aligned} stp(v) &= 0, \quad v < 0 \\ stp(v) &= 1, \quad v \ge 0 \end{aligned} \tag{4}$$

The standard Hewlett Packard model [8] could be fully described by (2) and (3). It has high accuracy and could correctly represent tantalum oxide memristor behavior in electric field [8]. Its main disadvantages according to SPICE realization are the use of a non-differentiable modulus function in (2) and a non-continuous step function in (3) [9]. The simulation time to obtain the current-voltage relationship of for this model was $t_1$ = 0.0499 s.

### B. Improvement of the standard HP Model [9]

The main improvement presented in [9] is the replacement of the non-differentiable modulus function and the step function by their continuous and differentiable analogues [9]. The analog of the step function is [9], [10]:

$$f_{st}(v) = \frac{1}{1+\exp(k\,v)} \tag{5}$$

where *k* is a constant with a negative value. The analogue of the standard modulus function in (2) is [9], [10]:

$$k(v) = v\left[\frac{1}{1+\exp(-\rho v)} - \frac{1}{1+\exp(\rho v)}\right] \tag{6}$$

The main advantage of this improved model is the lack of convergence problems in SPICE environment due to the introduced continuous and differentiable functions [9], [10]. The disadvantage of this model is its higher complexity. The simulation time to obtain the current-voltage relationship of for this model was $t_2$ = 0.0175 s.

### C. The Proposed Modified Memristor Model

The proposed modification of the standard HP memristor model contains three basic replacements: first, the term $a\exp\left(b\sqrt{|v|}\right)$ in (2) is replaced by $hv^2$, which has almost the same values in the voltage interval form -2 to +2 V and the mean squared error between these expressions is about 3 %; second, the step function in (3) is substituted by a differentiable analogue *g = (tanh(kv)+1)/2*; third, a modified Biolek window function *f(x,i)* with increased nonlinearity and a differentiable step function component is applied in order to bound the state variable in [0,1] [13], [14]. The proposed model is described by (7), (8) and (9):

$$i = v\,G_{eq} = v\left[G\,x + h\,v^2(1-x)\right] \tag{7}$$

$$\frac{dx}{dt} = \left[\begin{aligned} & B\sinh\left(\frac{v}{\sigma_{ON}}\right)\exp\left(\frac{v\,i}{\sigma_P} - \frac{x^2}{{x_{ON}}^2}\right)g(v) + \\ & + A\sinh\left(\frac{v}{\sigma_{OFF}}\right)\exp\left(-\frac{{x_{OFF}}^2}{x^2}\right) \\ & \exp\left(\frac{1}{1+\beta\,i\,v}\right)g(-v) \end{aligned}\right] f(x,i) \tag{8}$$

$$f(x,i) = \frac{\left\{1-\left[x-g(-i)\right]^{2p}\right\} + m\left[\sin^2(\pi x)\right]}{1+m} \tag{9}$$

The simulation time to obtain the current-voltage relationship of for this model was $t_3$ = 0.0158 s.

## III. TUNING THE PROPOSED MEMRISTOR MODEL

The modified memristor model proposed in this research and presented by (7), (8) and (9) contains many tuning parameters and coefficients. It is adjusted according to experimental current-voltage relationship and using a methodology for varying the coefficients' values till reaching global minimum of the mean square error [15], [16]. An algorithm for model parameters' estimation in MATLAB – Simulink environment [21] is also applied.

The applied methodology for optimizing the memristor model's adjustment is based on varying the fitting parameters and searching for a global minimum of the root mean square error between the experimental and the simulated current-voltage relationships. At each step of the algorithm one of the model's parameters is altering with a constant increment [16]. The corresponding root mean square error between the experimental and the obtained simulated current-voltage relationships is calculated. The other model's fitting parameters are changing as well. During this process a visual observation of the obtained current-voltage relation and its closeness to the experimental one is made. The condition for stopping the procedure is minimizing the root mean square error [15]. Extra experiments are made around the obtained optimal values of the model’s parameters using reduced steps for their varying.

The optimal model’s parameters are also estimated using the least squares algorithm in Simulink Design Optimization Toolbox in MATLAB environment [21]. All the applied signals are previously sampled. For input signal the memristor voltage $u_1$ is chosen. The experimentally measured output signal is denoted by $i_{mes}$. The calculated output signal is the obtained value of the memristor current $i_{calc}$. The cost function $S_{cost}$ is presented as a summation of the squares of the differences between the calculated and the experimentally measured output signal's values [15], [16]:

$$S_{\cos t} = \sum_{k=1}^{N} \left[ i_{calc}\left(k\right) - i_{mes}\left(k\right) \right]^2 \qquad (10)$$

where $N$ is the number of signals’ samples. It should be mentioned that it is continuously differentiable, because the calculated value of the current is continuously differentiable function. The criterion for stopping the algorithm is minimization the cost function [16].

The optimal values of the estimated model’s parameters are: $k = 400$, $A = 0.05 \cdot 10^{-10}$, $B = 0.3$, $\sigma_p = 3.7 \cdot 10^{-5}$, $\sigma_{off} = 1.9 \cdot 10^{-2}$, $\sigma_{on} = 4.5 \cdot 10^{-1}$, $G = 4.5 \cdot 10^{-2}$, $x_{off} = 9.99 \cdot 10^{-1}$, $x_{on} = 6 \cdot 10^{-2}$, $a = 7.2 \cdot 10^{-6}$, $\beta = 200$, $b = 4.7$, $x_0 = 0.19$, $p = 10$, $m = 0.1$, $h = 0.011$. The derived simulated current-voltage relationship [21] and the experimental one are presented in Fig. 2. The obtained error between them is 3.7 %.

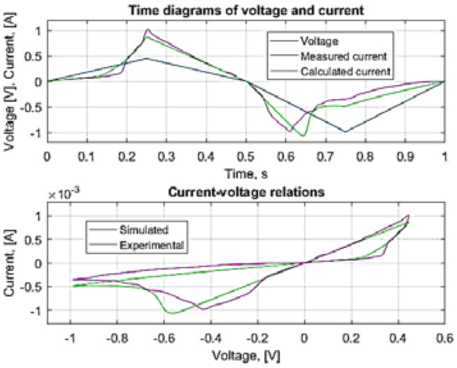

Fig. 2 a) Time diagrams of the voltage signal used for the memristor investigation, the experimental and the calculated current; b) Experimental and simulated current-voltage relationships

## IV. PSPICE LIBRARY MEMRISTOR MODEL

According to the proposed modified and improved mathematical model of tantalum oxide memristor, expressed by equations (7), (8) and (9), a PSpice [17] library model is created. For realization the mathematical operations according to the proposed model the standard functional blocks in ORCAD Capture are applied. Follows the net-list of the generated PSpice memristor library model.

```
.subckt tao_memristor a c
R_R1         C 0  1g
R_R2         N00302 C  1meg
E_DIFF1         U 0 VALUE {V(N00302,C)}
V_CONST1         A 0 DC 0.05e-10
V_CONST2         B 0 DC 3e-1
V_CONST3         SIGMAP 0 DC 3.7e-5
V_CONST4         SIGMAON 0 DC 4.5e-1
V_CONST5         G 0 DC 8e-2
V_CONST6         XOFF 0 DC 9.99e-1
V_CONST7         SIGMAOFF 0 DC 1.9e-2
V_CONST8        XON 0 DC 6e-2
V_CONST9          P 0 DC 4
V_CONST10        BETA 0 DC 500
V_CONST11        M 0 DC 0.1
V_CONST12        K 0 DC 400
V_CONST13        H 0 DC 0.014
G_ABMII1        A C VALUE { (V(IMEMR))    }
E_ABM3        N02740 0 VALUE { (V(U))*(1/V(sigmaon))    }
E_MULT1        N02559 0 VALUE {V(N02509)*V(N02798)}
E_ABM4   N02509 0 VALUE {+  exp ((V(U)) *V(imemr) * (1/V(sigmap)))
-((V(x)/V(xon))  *(V(x)/  V(xon)))        }  E_MULT2  N02678  0  VALUE
{V(GPLUS)*V(N02559)}
     E_ABM5     GPLUS     0     VALUE     {+((exp((V(U))*V(k))-exp(-
(V(U)*V(k))))/((exp((V(U))*V(k))+exp(-((V(U))*V(k)))))+1)/2 +     }
     E_ABM6          N02798  0  VALUE  {  V(B)*((exp(V(N02740))-exp(-
V(N02740)))/2)    } E_SUM1 N03129 0 VALUE {V(N03712)+V(N02678)}
E_ABM7  N02937 0 VALUE { exp(1/(1+V(beta)*V(imemr)*(V(U))))
     E_MULT3        N02986 0 VALUE {V(GMINUS)*V(N02937)}
     E_ABM8    WIND    0    VALUE    {+    ((1-pwrs((V(x)-V(gminus)),
(2*V(p))))+((V(M))*(sin(3.14*V(x)))*((sin(3.14*V(x))))))/(1+(V(M)))      +
} E_MULT4        N03167 0 VALUE {V(WIND)*V(N03129)}
     X_INTEG1    N03167 X SCHEMATIC1_INTEG1
     E_ABM9        IMEMR 0 VALUE {+  (V(U))*((V(G)*V(x)+(V(h)*(1-
V(x)))*((V(U)))*((V(U)))))        }  E_ABM10    GMINUS  0  VALUE  {+
((exp((V(N03507))*V(k))-exp(-
((V(N03507))*V(k))))/((exp((V(N03507))*V(k))     +     exp(-((V(N03507))
*V(k))))) +1)/2+    } E_GAIN1        N03507 0 VALUE {-1 * V(U)}
     E_ABM11                  N03688  0  VALUE  {+   (V(A)*  ((exp(V(U)/
V(sigmaoff))-exp(-V(U)/V(sigmaoff)))/2))*(exp(-(V(xoff)/ V(x))*((V(xoff)/
V(x))))) +    }  E_MULT5  N03712  0  VALUE  {V(N03688)*V(N02986)}
R_R3        A N00302  10  .ends
```

The created PSpice memristor model is successfully tested for several frequencies and amplitudes of the applied voltage. The derived current-voltage relationships are presented in Fig. 3. During the computer simulations no convergence problems have been observed.

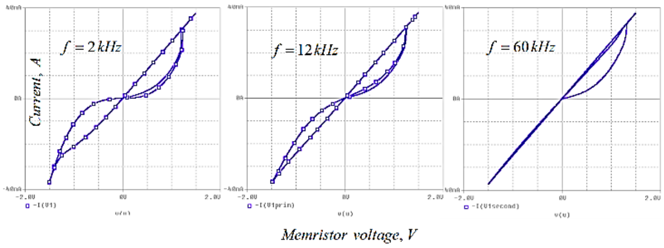

Fig. 3 Current-voltage relationships of the proposed memristor model for several different frequencies and amplitudes of the voltage derived in PSpice

## V. A MEMRISTOR-BASED NEURAL NETWORK

The considered memristor model is applied in a simple multilayer neural network [18] with memristor-resistor synapses [19] presented in Fig. 4. It is used for function fitting [18]. The input signal is: $x = 4.2 \exp(1.9\, t) + \exp(-6\, t)\, 5.5 \sin(2\, \pi\, 5\, t + \Psi_x)$; the desired signal is: $d = 1.1 \exp(1.99\, t)$. The network contains five neurons in the hidden layer [18]. The applied synapses are memristor-based [19]. The schematic of a synapse is presented in Fig. 5. The synapse is realized in bridge topology [19]. It contains two memristors connected in anti-parallel and two resistors [19]. The synaptic weight of the applied memristor-based synapse could be expressed as follows:

$$w = \frac{v_{out}}{v_{in}} = \frac{M_1}{R_3 + M_1} - \frac{M_2}{R_4 + M_2} \qquad (11)$$

The change of synaptic weight is based on applying external voltage pulses and altering the memristors’ states and resistances, respectively [19]. The operation of the considered neural network is based on learning and synaptic weight update [19]. The input signal and the desired output signal are presented in Fig. 6. After the learning stage the appropriate synaptic weights are established and the output signal matches to the desired signal. In the initial moment of the analysis all the synaptic weights are zeros. After finishing the learning process they are: for the biases of the input layer $\boldsymbol{b_1}$ = [-0,6279; -0,6382; 0,0956; 0,8670; 0,5598]; for the biases of the hidden layer $\boldsymbol{b_2}$ = [0.5926]; for the input layer $\boldsymbol{v}$ = [0,9344; 0,9532; 0,4036; 0,9599; 0,8042]; for the hidden layer $\boldsymbol{w}$ = [0,6355; 0,6908; -0,1234; 0,6397; -0,6575].

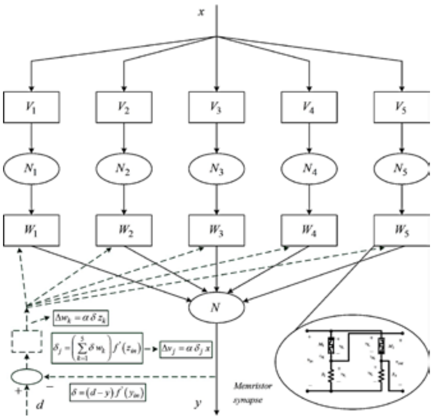

Fig. 4 A memristor-based neural network for function fitting

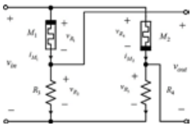

Fig. 5 A synapse based on memristors

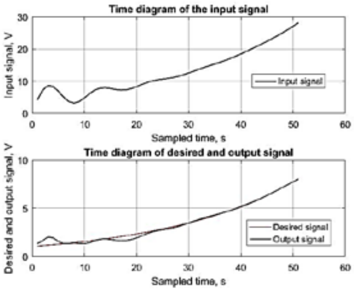

Fig. 6 Time diagrams of the input, output and the desired signals of the memristor-based neural network

## VI. CONCLUSION

The applied nonlinear tantalum oxide memristor model with a modified window function is successfully fitted to experimental current-voltage relationships of $Ta_2O_5$ – based memristor nanostructures. It contains many tuning parameters and a good accuracy. The proposed model's modification has simplified structure and increased operating speed with respect to the original HP model. The applied modified window function limits the state variable and represents the boundary effects for hard-switching mode. A PSpice library model is created according to the model's equations. It was successfully simulated and no convergence problems have been observed. The modified model is successfully tested in a simple multilayer memristor-based neural network for function fitting. The memristor model's capability for operation in complex memristor-based electronic schemes is established.

## REFERENCES

[1] D. Strukov, G. Snider, D. Stewart, R. S. Williams, "The missing memristor found," *Nature Letters*, Vol 453, 2008, doi:10.1038/nature06932, pp. 80 – 83.

[2] F. Corinto, P. Civalleri, L. Chua, "A Theoretical Approach to Memristor Devices," *IEEE J. on Emerging and Selected Topics in Circ. and Syst.* Vol. 5, 2015, DOI: 10.1109/JETCAS.2015.2426494, pp. 123 – 132.

[3] A. Torrezan, J. Strachan, G. Medeiros-Ribeiro, R. S. Williams, "Sub-nanosecond switching of a tantalum oxide memristor," *IOP Publ. Nanotech.*, 22 (2011) doi:10.1088/0957-4484/22/48/485203, pp. 1–7.

[4] A. Ascoli, R. Tetzlaff, Z. Biolek, Z. Kolka, V. Biolkovà, D. Biolek, "The Art of Finding Accurate Memristor Model Solutions," *IEEE J. Emerg. Sel. Top. Circuits Syst*., 5, 2015, pp. 133–142.

[5] F. Chien Chiu, "A Review on Conduction Mechanisms in Dielectric Films," *Hindawi Publishing Corporation, Advances in Materials Science and Engineering*, Vol. 2014, Article ID 578168, 18 pages, http://dx.doi.org/10.1155/2014/578168.

[6] J. Pacheco, D. Perry, D. Hughart, M. Marinella, E. Bielejec, "Electroforming-Free $TaO_x$ Memristors using Focused Ion Beam Irradiations," *Springer, Appl. Phys*. A, 2018, https://doi.org/10.1007/s00339-018-2041-3.

[7] P. Mickel, A. Lohn, B. Choi, J. Yang, M. Zhang, M. Marinella, C. James, R. S. Williams, "A physical model of switching dynamics in tantalum oxide memristive devices," *Applied Physics Letters*, Vol. 102, Issue 22, 223502 (2013), pp. 1 – 5.

[8] J. Strachan, A. Torrezan, F. Miao, M. Pickett, J. Yang, W. Yi, G. Medeiros-Ribeiro, R. S. Williams, "State Dynamics and Modeling of Tantalum Oxide Memristors," *IEEE Transactions on Electron Devices*, Vol. 60, No. 7, July 2013, pp. 2194 - 2202, DOI 10.1109/TED.2013.2264476.

[9] A. Ascoli, R. Tetzlaff, L. Chua, "Robust Simulation of a TaO Memristor Model," *Radioengineering*, Vol. 24, No. 2, June 2015, DOI: 10.13164/re.2015.0384, pp. 384 – 392.

[10] V. Ntinas, A. Ascoli, R. Tetzlaff, G. Sirakoulis, "Transformation techniques applied to a TaO memristor model to enable stable device simulations," *IEEE Proceedings of 2017 European Conf. on Circuit Theory and Design*, DOI: 10.1109/ECCTD.2017.8093286, pp. 1 – 4.

[11] C. Yakopcic, T. Taha, G. Subramanyam, R. Pino, "Memristor SPICE Modeling," *Advances in Neuromorphic Memristor Science and Applications*, DOI 10.1007/978-94-007-4491-2_12, pp. 211 – 244.

[12] C. Yakopcic, T. Taha, G. Subramanyam, R. Pino, "Memristor SPICE Model and Crossbar Simulation Based on Devices with Nanosecond Switching Time," *Proceedings of IEEE Int. Joint Conf. on Neural Networks*, Dallas, Texas, USA, August 4-9, 2013, pp. 464 – 470.

[13] V. Mladenov, S. Kirilov, "A Nonlinear Drift Memristor Model with a Modified Biolek Window Function and Activation Threshold," *MDPI Electronics*, 2017, 6(4), 77; doi:10.3390/electronics6040077, pp.1-15.

[14] V. Mladenov, "Advanced Memristor Modeling - Memristor Circuits and Networks," *MDPI Basel, Switzerland*, ISBN 978-3-03897-104-7 (Hbk), https://doi.org/10.3390/books978-3-03897-103-0, 2019.

[15] S. Chen, S. Billings, W. Luo, "Orthogonal least squares methods and their application to non-linear system identification", *International Journal of Control, Taylor & Francis,* 1989, https://doi.org/10.1080/00207178908953472, pp. 1873 - 1896.

[16] J. Zhou, J. Shi, "A comprehensive multi-factor analysis on RFID localization capability," *Advanced Engineering Informatics*, 25 (2011), doi:10.1016/j.aei.2010.05.006, pp. 32–40.

[17] M. Rashid, "Introduction to PSpice using OrCAD for circuits and electronics," *Prentice Hall*, 3rd ed., 2004, ISBN 0-13-101988-0.

[18] L. Fausett, "Fundamentals of Neural Networks," *Prentice Hall*, 1994, ISBN 0130422509.

[19] V. Mladenov, "Synthesis and Analysis of a Memristor-Based Artificial Neuron," *IEEE, VDE Proceedings of CNNA 2018, The 16th International Workshop on Cellular Nanoscale Networks and their Applications*, Print ISBN: 978-3-8007-4766-5, pp. 1 – 4.

[20] S. Choi, P. Sheridan, Wei D. Lu, "Data Clustering using Memristor Networks," 2015, *Nature, Scientific Reports* | 5:10492 | DOI: 10.1038/srep10492, pp. 1 – 10.

[21] W. Palm, "Introduction to MATLAB for engineers," *McGraw-Hill, New York*, 2011, ISBN 978-1-259-01205-1.