

# Nonlinear pitch control of a large scale wind turbine by considering aerodynamic behavior of wind

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**Abstract**— In this research, nonlinear sliding mode pitch control of a wind turbine has been investigated by considering aerodynamic nonlinearities. For modeling aerodynamic interaction between the wind and the drive-train system, blade element momentum theory is used by considering Prandtl's tip loss factor and Glauert correction. Finally, the two-degrees of freedom model of the drive-train is extracted and the sliding mode approach is examined for regulating the output power into its nominal value by controlling the pitch angle. The implementation of the above proposed control law in its related electronic circuit of the wind turbine will be considered as the future stage of the current research.

**Keywords**—Wind turbine control, aerodynamic nonlinearities, sliding mode control

## I. INTRODUCTION

The significance of renewable energies is incredible in the new decades. The criticism of global warming which is directly related to the carbon dioxide production and lack of fossil fuels are the main reasons for the development of the renewable energy industries [1]. Furthermore, enlargement of the renewable energy industries is one of the main research plans in developed countries. In comparison with the other sources of renewable energy, wind energy is experiencing one of the most progressed rates [2]. There are four operational regions for horizontal wind turbines. In region 1, the wind velocity is lower than the cut-in wind speed and wind energy is not capable to overcome the resistance forces to move the blades. In region 2, the wind velocity is bigger than the cut-in wind speed and lower than the rated one and the goal is to absorb the mechanical power as much as it is possible in the safety condition. In region 3, the wind velocity is bigger than the rated wind speed and lower than the cut-out wind velocity. In this region, the output power must track its nominal value for safety reasons and finally, in region 4, when the wind velocity is bigger than the cut-out one, the wind turbine must be stopped for safety reasons [3]. One of the main sources of nonlinearities in the wind turbine systems is the aerodynamic description. Many research works, prefer to use a nonlinear surface for describing the power coefficient of the wind turbine as a function of the tip speed ratio and blade pitch angle [4]. The main problem of this

approach is losing some data information in the curve fitting process. In reference [5], the pitch angle control is described by considering the aerodynamic nonlinearities and using the hydraulically actuated system. In region 3, one of the main sources of nonlinearities in the wind turbine systems is the aerodynamic description. Two PID fuzzy systems (feedback and feedforward) are used for generating the desired pitch angle and tracking that. Results are obtained in different wind speed profiles (step and random) and in faulty conditions. Although the aerodynamic model is derived and validated, the drivetrain dynamic of the wind turbine is ignored. Furthermore, the cascading of the control system is not always allowable. In this paper, the problem of wind turbine control in region 3 has been considered by considering the nonlinearities in aerodynamic modeling. In the first step, the aerodynamic behavior of the interaction between the blades and wind is needed to be obtained. Blade Element Momentum (BEM) theory with Prandtl and Glauert correction is used for obtaining the aerodynamic forces and torques. In order to be confident of the consistency of the model, the power coefficient of NREL 5MW is derived as a function of the tip speed ratio in the zero pitch angle (similar simulations can be extracted for different pitch angles) and the results are validated by FAST code which is widely used for simulating the behavior of onshore and offshore wind turbines. Finally, in order to track the rated power, the sliding mode approach is used which is known for its robustness ability. The results demonstrated that the sliding mode approach performs extraordinary.

## II. BLADE ELEMENT MOMENTUM THEORY FOR OBTAINING THE AERODYNAMIC CHARACTRISTICS

The NREL 5MW onshore wind turbine consists of 8 different airfoil kinds and 17 different sections along the blade. Wind turbines extract the kinetic energy of the wind into the benefit of mechanical energy. Therefore, there is a reduction in the wind speed in the rotor and in the wake in comparison of upstream wind speed ( $V_0$ ) which is described by induction factor  $\alpha$  as [6]:

$$u_a = V_0 (1-a) \quad (1)$$

Similarly, from the blade perspective, because of the rotation of the blade, there is a lateral composition of the wind speed ( $u_l$ ) which can be written as [6]:

$$u_l = r\omega(1+a') \quad (2)$$

where  $r$  is the radial position of the annular element,  $\omega$  is the rotational speed of the wind turbine and  $a'$  is the lateral induction factor.

The relative magnitude wind velocity ( $V_{rel}$ ) which is observed by the blade perspective is obtained as:

$$V_{rel} = V_0 \sqrt{\left(1-a\right)^2 + \left(\frac{r\omega_r}{V}(1+a')\right)^2} \quad (3)$$

Also, it is easy to see the angle of the relative wind velocity with respect to the rotor plane:

$$\tan(\phi) = \frac{V_0}{r\omega_r} \frac{1-a}{1+a'} \quad (4)$$

According to the basics of the 2-D aerodynamic foundations, the lift force is perpendicular to the relative wind velocity and the drag force is parallel to that. It is common to normalize the lift and drag forces into the lift and drag coefficients ( $C_L(\alpha)$ ,  $C_D(\alpha)$ ) which are defined as:

$$f_L = \frac{\rho c}{2} V_{rel}^2 C_L(\alpha) \quad (5)$$

$$f_D = \frac{\rho c}{2} V_{rel}^2 C_D(\alpha) \quad (6)$$

The lift and drag coefficients are functions of the Reynolds number, Mach number, and angle of attack. The wind field is incompressible (Mach number is lower than 0.3) which means that the lift and drag coefficients are independent of the Mach number. The dependency of the airfoil properties into the Reynolds number varies in different airfoils. In the case of the NREL 5MW turbine, there is a weak dependency for 8 airfoils properties related to Reynolds number. The angle of attack is the angle between the relative velocity and the chord line. The angle of attack of each element can be obtained as follow:

$$\alpha = \phi - \beta \quad (7)$$

where  $\phi$  is the angle between the wind relative speed and rotational plane and  $\beta$  is the pitch angle. By considering the normal and tangential forces to the rotational plane, these forces can be obtained as [6]:

$$\begin{aligned} p_N &= f_L \cos \phi + f_D \sin \phi \\ p_T &= f_L \sin \phi - f_D \cos \phi \end{aligned} \quad (8)$$

These forces can be normalized by the term  $\frac{1}{2} \rho V_{rel}^2 c$ , which yields:

$$\begin{aligned} C_N &= \frac{p_N}{\frac{1}{2} \rho V_{rel}^2 c} \\ C_T &= \frac{p_T}{\frac{1}{2} \rho V_{rel}^2 c} \end{aligned} \quad (9)$$

According to [6], it is readily to see the following equation:

$$\begin{aligned} V_{rel} \sin \phi &= V_0 (1-a) \\ V_{rel} \cos \phi &= \omega r (1+a') \end{aligned} \quad (10)$$

The solidity  $\sigma$  is defined as the fraction between the annular area in the control volume which is observed by blades [6]:

$$\sigma = \frac{C(r)N}{2\pi r} \quad (11)$$

where  $N$  denotes the number of blades,  $C(r)$  is the length of the chord in each section of the blade as a function of local position of the section and  $r$  is the local position. The normal force and the torque to the control volume with thickness  $dr$  can be obtained as:

$$\begin{aligned} df_T &= N p_N dr \\ dM &= r N p_T dr \end{aligned} \quad (12)$$

By the combination of Eqs. (10) and (12), Eq. (12) can be written as:

$$\begin{aligned} df_T &= \frac{1}{2} \rho N \frac{V_0^2 (1-a)^2}{\sin^2 \phi} c C_n dr \\ dM &= \frac{1}{2} \rho N \frac{V_0^2 (1-a) \omega r (1+a')}{\sin \phi \cos \phi} c C_T dr \end{aligned} \quad (13)$$

Similarity, from ideal rotor theory, the thrust and torque can be computed as:

$$\begin{aligned} df_T &= 4\pi r \rho V_0^2 a (1-a) dr \\ dM &= 4\pi r^3 \rho V_0^3 \omega (1-a) a' dr \end{aligned} \quad (14)$$

By the combination of Eqs. (12) and (11), the parameters  $a'$  and  $a$  can be obtained as [6]:

$$\begin{aligned} a &= \frac{1}{1 + \frac{4 \sin^2 \phi}{\sigma C_n}} \\ a' &= \frac{1}{\frac{4 \sin \phi \cos \phi}{\sigma C_n} - 1} \end{aligned} \quad (15)$$

#### A. Prandtl's Tip loss Factor and Glauert correction for high values of $a$

According to the BEM theory, there are two basic assumptions which are written below:

- There is no radial dependency which means that each airfoil does not have any sense with another
- There is no change in the aerodynamic properties in each annular element which is true for a rotor with an infinite number of blades.

In the above processor, it is assumed that the rotor has infinite number of blades and the Prandtl's Tip loss factor corrects this assumption. Prandtl derived a correction variable  $F$  to correct Eq. (13) which is written below:

$$df_T = \frac{1}{2} \rho N \frac{V_0^2 (1-a)^2}{\sin^2 \phi} c C_n F dr$$

$$dM = \frac{1}{2} \rho N \frac{V_0^2 (1-a) \omega r (1+a)}{\sin \phi \cos \phi} c C_T F dr$$
(16)

where  $F$  is computed as [6]:

$$F = \frac{2}{\pi} \cos^{-1} \left( e^{-f} \right), f = \frac{N}{2} \frac{R-r}{r \sin \phi}$$
(17)

By this assumption, the axial induction factors  $a$  is corrected as:

$$a = \frac{1}{1 + \frac{4F \sin^2 \phi}{\sigma C_n}}, a' = \frac{1}{\frac{4F \sin \phi \cos \phi}{\sigma C_n} - 1}$$
(18)

when the axial induction flow factor is larger than 0.4, the classical BEM theory is invalid. According to Glauert [6], the following correction is needed:

$$C_T = \begin{cases} 4a(1-a)F & a \leq a_c \\ 4(a_c^2 + (1-2a_c)a)F & a \geq a_c \end{cases}$$
(19)

where  $a_c$  is approximately 0.2.

### III. VALIDATING THE AERODYNAMIC MODEL BY FAST CODE

In order to validate the aerodynamic model, the power coefficient for different pitch angles must be obtained as a function of tip speed ratio. By following the steps of the previous algorithm, the power coefficient can be obtained as:

$$C_P(\lambda, \beta) = \frac{P_r}{\frac{1}{2} \rho \pi R^2 V^3}$$
(20)

In Fig. (1), the power coefficient in the pitch angle of zero has been obtained as a function of tip speed ratio and compared to the results of aeroelastic code FAST. The results demonstrate that the aerodynamic model has been obtained in a perfect way.

### IV. WIND SPEED MODELING

Usually, wind speeds are measured by anemometers at a point by considering the sampling frequency. As time series, wind

speed can be expressed by discreet Fourier time series which is written below [6]:

$$u(t) = \bar{u} + \sum_{n=1}^{\frac{N}{2}} a_n \cos(\omega_n t) + b_n \sin(\omega_n t)$$

$$= \bar{u} + \sum_{n=1}^{\frac{N}{2}} \sqrt{a_n^2 + b_n^2} \cos(\omega_n t - \varphi_n), \omega_n = \frac{2\pi n}{T}$$
(21)

where  $\bar{u}$  is the mean wind velocity,  $T$  is the period of the signal,  $N$  is the size of the time series and  $\varphi_n$  is the uniform random generated number. It is worthy to note that in order to prevent the distortion of the signal, the sampling frequency must be greater than two times of the maximum frequency of the signal which is called the aliasing phenomena. That is the reason why the summation in Eq. (21) starts from the frequency 0 and ends to the frequency  $\frac{N}{2}$ . It can prove that the time series in Eq. (21) can be expressed as follow [6]:

$$u(t) = \bar{u} + \sum_{n=1}^{\frac{N}{2}} \sqrt{\frac{2PSD(f)}{T}} \cos(\omega_n t - \varphi_n)$$
(22)

where  $PSD(f)$  is the power spectral density function which is a function of frequency (in Hz). There are many power spectral functions that are tried to be used by the researches for modeling the wind speed profile by considering the atmospheric boundary layer. One of these is called the Kaimal spectrum; that is expressed as:

$$PSD(f) = \frac{I^2 V_{10\text{min}} l}{\left( 1 + 1.5 \frac{f \cdot l}{V_{10\text{min}}} \right)^{\frac{5}{3}}}$$
(23)

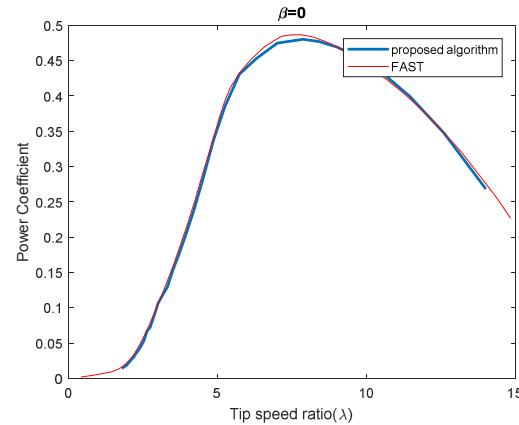


Fig.1 power coefficient as a function of tip speed ratio in constant pitch angle 0: a comparison between FAST aeroelastic code and proposed algorithm

where  $I = \sigma_{V_{10\text{min}}} / V_{10\text{min}}$  is called the turbulence intensity.  $V_{10\text{min}}$  is the 10-minute averaged wind speed and  $l$  is the length scale.  $l = 20h$  for  $h < 30$  and  $l = 600$  for  $h > 30$ .

## V. SLIDING MODE CONTROLLER DESIGN FOR REGULATING THE POWER

The drive-train dynamics of the wind turbine contains the rotor (the three blades of the wind turbine), low-speed shaft, gearbox and high-speed shaft which can be obtained as [7]:

$$\left\{ \begin{array}{l} T_a(\omega_r, \beta, v) - K_{ls}(\theta_r - \theta_{ls}) \\ \dot{\omega}_r = \frac{-D_{ls}(\omega_r - \omega_{ls}) - D_r \omega_r}{j_r} \\ -T_e n_g + K_{ls}(\theta_r - \theta_{ls}) \\ \dot{\omega}_{ls} = \frac{+D_{ls}(\omega_r - \omega_{ls}) - D_g n_g^2 \omega_{ls}}{j_g n_g^2} \\ \dot{\beta} = -\frac{1}{\tau}(\beta - u) \end{array} \right. \quad (24)$$

where  $\omega_r$  is rotor speed,  $\omega_{ls}$  is speed of low speed shaft,  $\theta_r$  is rotational angle of rotor,  $\theta_{ls}$  is rotational angle of low speed shaft,  $K_{ls}$  is stiffness factor of low speed shaft,  $D_{ls}$  is damping factor of low speed shaft,  $j_r$  is rotor inertia,  $j_g$  is generator inertia,  $D_r$  is external damping factor of rotor,  $D_g$  is external damping factor of generator,  $n_g$  is gearbox ratio and  $T_e$  is generator torque. In the next, sliding mode approach is considered for regulating the pitch dynamic by introducing the sliding surface and Lyapunov function as below:

$$S = \dot{e} + \alpha_1 e, v = \frac{1}{2} S^2, \dot{v} = -\eta |S| \quad (25)$$

After some straightforward computations we reach to:

$$u = \begin{pmatrix} J_r(-\eta \text{sign}(S) - \alpha_1 e - \alpha_2) \\ +K_{ls}(\omega_r - \omega_{ls}) \\ +D_{ls}(\dot{\omega}_r - \dot{\omega}_{ls}) - \frac{\partial T_a}{\partial \omega_r} \dot{\omega}_r \end{pmatrix} \frac{\tau}{\partial T_a} + \beta \quad (26)$$

In Figs. (2) and (3), the output power and pitch angle are obtained respectively which means the output power has tracked its nominal value.

## VI. CONCLUSION

In this paper, the problem of wind turbine control in region 3 has been investigated by considering the nonlinear aerodynamic model of the wind-blade interaction. By using the

sliding mode approach, the output power tracks its nominal value perfectly.

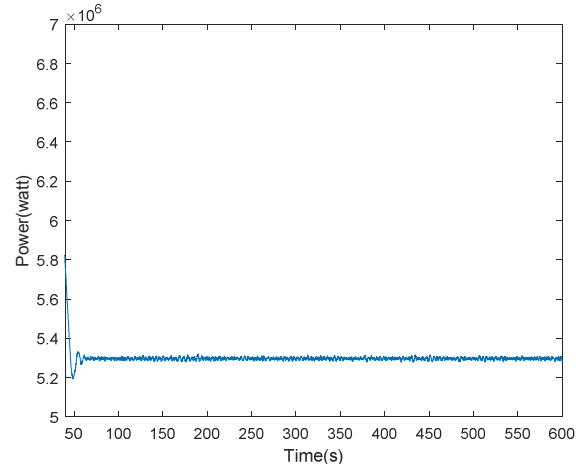


Fig.2 output power time history as a function of time.

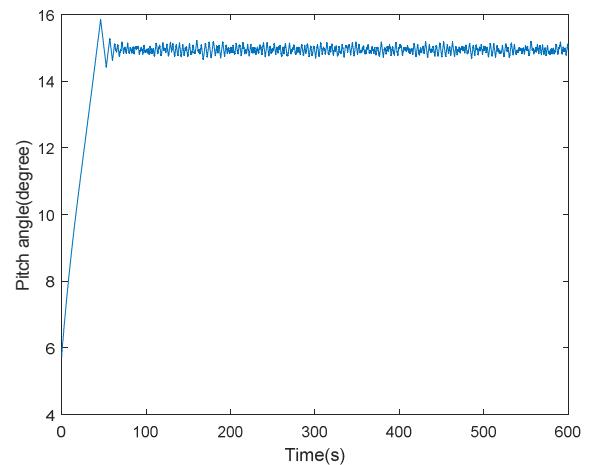


Fig.3 pitch angle variation in order to track the nominal value of power

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