# A Novel Chaotic System with Application to Secure Communications

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***Abstract*—In this work, a novel three dimensional chaotic system with is proposed. The system has no linear terms and a line equilibrium, so it belongs to the category of systems with hidden attractors. The system's dynamical behavior is analysed through its bifurcations diagrams and maximum Lyapunov exponent diagram. Then, the system is applied to the problem of secure communications using the Symmetric Chaos Shift Keying modulation method.**

***Index Terms*—Chaos, hidden attractor, secure communications, CSK modulation**

## I. Introduction

Since its introduction in the 1960's, chaos theory has been integrated in almost all scientific disciplines, from physics to engineering, biology, medicine, chemistry and economics. Due to their high sensitivity to parameter changes and initial conditions, chaotic systems have been proven to be very useful in applications that require the utilization of systems with high complexity, like encryption [1], communications [2], [3], robotics [4], random number generators [5] and more [6].

Thus, there is a continuous interest in designing new chaotic systems, since their demand in applications is ongoing. To design new chaotic systems, researchers either consider modifying existing ones by adding more terms or modifying the existing ones, or design new ones by utilizing nonlinear functions that are documented to easily give chaotic behavior, like the hyperbolic sine function, the exponential function, the cubic power, or the absolute value, see for example [7]–[11] and also the references found therein.

In the collective effort to propose new chaotic systems, special attention is given to designing chaotic systems with hidden attractors [12]. An attractor is called hidden if its basin of attraction does not intersect with any open neighbourhood of an equilibrium. Systems with hidden attractors have gained a lot of attention over the last years due to their complex behavior, which also makes them attractive for applications. Some examples of recent works on novel systems with hidden attractors are [9], [10], [12]–[16].

Motivated by the above, this work proposes a chaotic system with a line equilibrium, which belongs to the category of hidden attractors. The proposed system has no linear terms, and two parameters. A dynamical analysis on the system is performed and its chaotic behavior is unmasked, by studying its bifurcation diagrams and maximum Lyapunov exponent diagram.

Furthermore, to showcase the implementability of the system to chaos related applications, the system is applied to the problem of digital secure communications. The problem under study here is to mask an information signal using a chaotic system, so as to safely transmit it from a transmitter unit to a receiver, where it will be reconstructed after appropriate signal processing [3], [17]. The method used here is the Symmetric Chaos Shift Keying (SCSK) modulation proposed in [16].

The rest of the paper is organised as follows: Section II presents the proposed chaotic system, along with its dynamical analysis. In Section III, the system is applied to the problem of secure communications. Section IV concludes the paper with

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a discussion on future goals.

## II. The Proposed Dynamical System

In this section, a novel 3-D chaotic system is proposed and its qualitative properties are presented in detail. The proposed system is described as:

$$\begin{cases} \dot{x} = yz \\ \dot{y} = x^3 - y^3 \\ \dot{z} = a|x| - by^3 - xy \end{cases} \tag{1}$$

where $x$, $y$, $z$ are the state variables and $a$, $b$ are positive parameters. System (1) is chaotic for a wide range of parameter values. For example, when parameters $a$ and $b$ take the values $a = 0.5$, $b = 0.1$, the Lyapunov exponents of the novel system are $L_1 = 0.0799$, $L_2 = 0$, $L_3 = -1.3683$, which indicate the chaotic behavior of the system, as it is presented in the phase portraits of Figs. 1 and 2. Furthermore system (1) is dissipative because the sum of the Lyapunov exponents is negative.

Also, the proposed system (1) is structurally a very simple 3-D dynamical system having only nonlinear terms and two independent parameters $(a, b)$. Furthermore, system (1) has a line of equilibrium points $(0, 0, z)$. So, it belongs to the recently discovered dynamical systems, which are known as systems with hidden attractors.

For the more detailed analysis of system's (1) dynamical behavior the system is investigated numerically by using the fourth order Runge-Kutta algorithm. The bifurcation diagram, which is a very useful tool from nonlinear theory, is used in order to analyze system's (1) behavior. This diagram is produced when the trajectory cuts the plane $x = 0$ with $dx/dt < 0$. In this way, the bifurcation diagram of variable $z$ versus the parameter $a$, for $b = 0.1$ reveals the richness of system's dynamical behavior (Fig. 3). It is clear that as the parameter $a$ is increased the system enters chaos through a period doubling route (see the enlargement figure for $a \in [0.04, 0.09]$), while the extended chaotic region is interrupted by small windows of periodic behavior. As an additional simulation, the bifurcation diagram of variable $y$ versus the parameter $a$, for $b = 0.1$ is shown in Fig. 4, from which the same dynamical behavior can be observed.

In Fig. 5 the diagram of the Maximal Lyapunov Exponent (MLE) versus the value of parameter $a$, by using the well-known Wolf *et al.* [18] algorithm, is depicted. As we can see, when the system has a periodic behavior, the MLE is equal to zero, while when it has chaotic behavior the system has a positive MLE, as it is expected according to theory. So, from Fig. 5 system's (1) chaotic behavior is found in six ranges of parameter $a$, i.e. $a \in [0.069, 0.080]$, $a \in [0.085, 0.139]$, $a \in [0.160, 0.281]$, $a \in [0.289, 0.543]$, $a \in [0.547, 0.566]$ and $a \in [0.573, 0.598]$, confirming the bifurcation diagram of Fig. 3. Also, as parameter $a$ is increases the value of the MLE also increases driving the system to a more complex chaotic behavior.

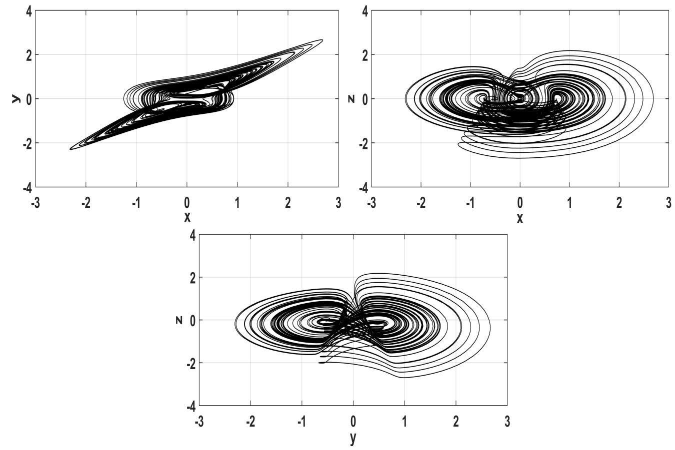

Fig. 1. Phase portraits of system (1), for $a = 0.5$ and $b = 0.1$.

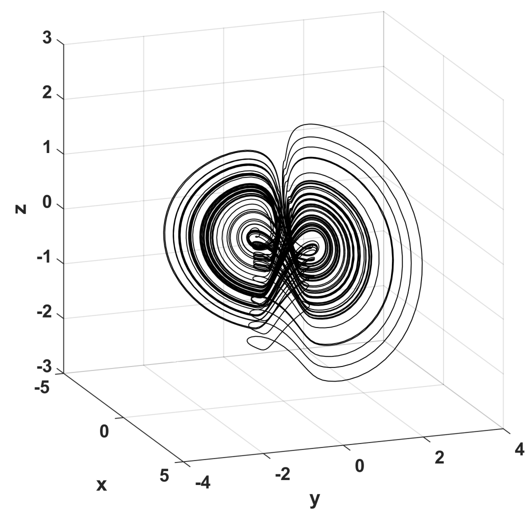

Fig. 2. 3D chaotic attractor of system (1), for $a = 0.5$ and $b = 0.1$.

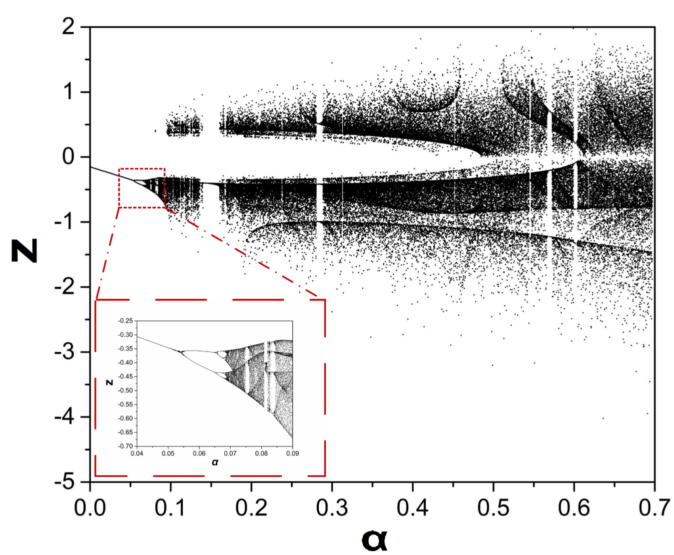

Fig. 3. Bifurcation diagram of $z$ versus $a$ for $b = 0.1$.

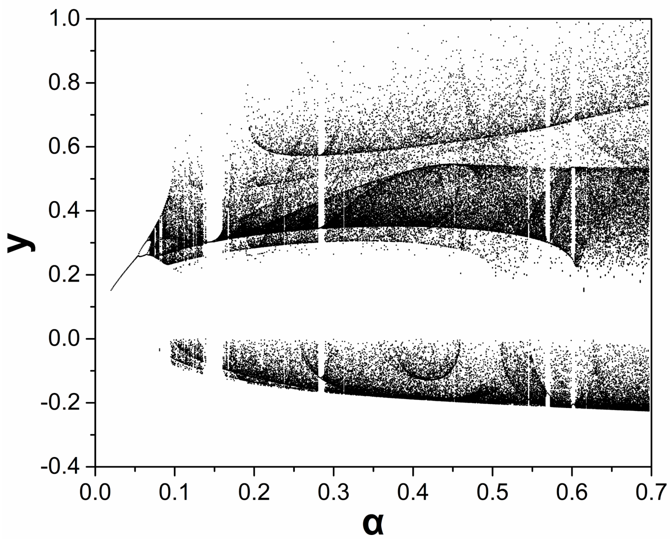


Fig. 4. (a) Bifurcation diagram of $y$ versus $a$ for $b = 0.1$.

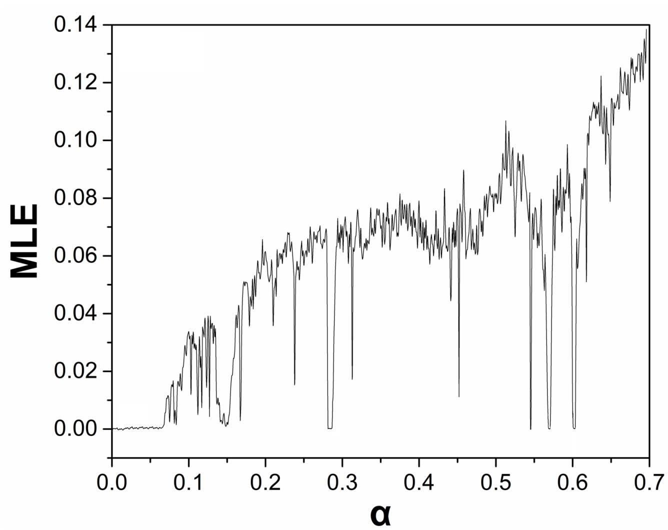


Fig. 5. Diagram of the maximal Lyapunov exponents (MLE) versus the parameter $a$, for $b = 0.1$.

## III. Application to Secure Communications

In this section, the proposed chaotic system is applied to the problem of secure communications using the Symmetric Chaos Shift Keying (SCSK) modulation method, which was applied in [16]. The system design is shown in Fig. 6. The system consists of a transmitter and a receiver unit.

In the transmitter unit, a given information signal is multiplied (modulated) with a chaotic signal generated by the chaotic system. The generated modulated signal is then transmitted to the receiver. In the simulations, the information signal is taken as a binary signal with a period of $1sec$. The chaotic signal used is the state $x$ of (1), with $a = 0.65$, $b = 0.1$. Of course, any other state, or a combination of them, can be used as the chaotic signal. In the transmission, it is also assumed that the channel is disturbed by an Additive White Gaussian Noise (AWGN). In the receiver unit, the received signal is multiplied by the chaotic signal generated by the same chaotic system as in the receiver, and the resulting signal is integrated in the correlator. So the bit energy of the signal is calculated and the bit information signal is reconstructed by passing the resulting signal through a threshold detector, with threshold set to 0.

The designed SCSK communication system was analysed under the AWGN channel. The retrieved signal is delayed by $0.5$ seconds due to the Sample/Hold block on the receiving unit. Therefore, there is a $0.5$ second delay between the transmitted information signal and the retrieved information signal on the receiving unit. So, the transmitted information signal was delayed by 0.5 seconds during the analysis. Fig. 7, shows the simulation of (a) the information signal, (b) the SCSK modulated signal, (c) the transmitted noisy signal and (d) the retrieved information signal, for $Eb/No = -10dB$. From Fig. 7(c) it is clear that the information signal cannot be estimated from the transmitted one.

## IV. Conclusions

In this work, a novel three dimensional chaotic system with no linear terms and two parameters was proposed. After appropriate analysis, it was shown that the system shows chaotic behavior for a wide range of parameters. To showcase the implementability of the proposed system to chaos related engineering applications, the system was applied to a secure communications design using the SCSK modulation method. Future aspects of this work would be an extensive BER performance comparison, the experimental implementation of the proposed design, and also the further application of the proposed system to pseudo random bit generators and image encryption.

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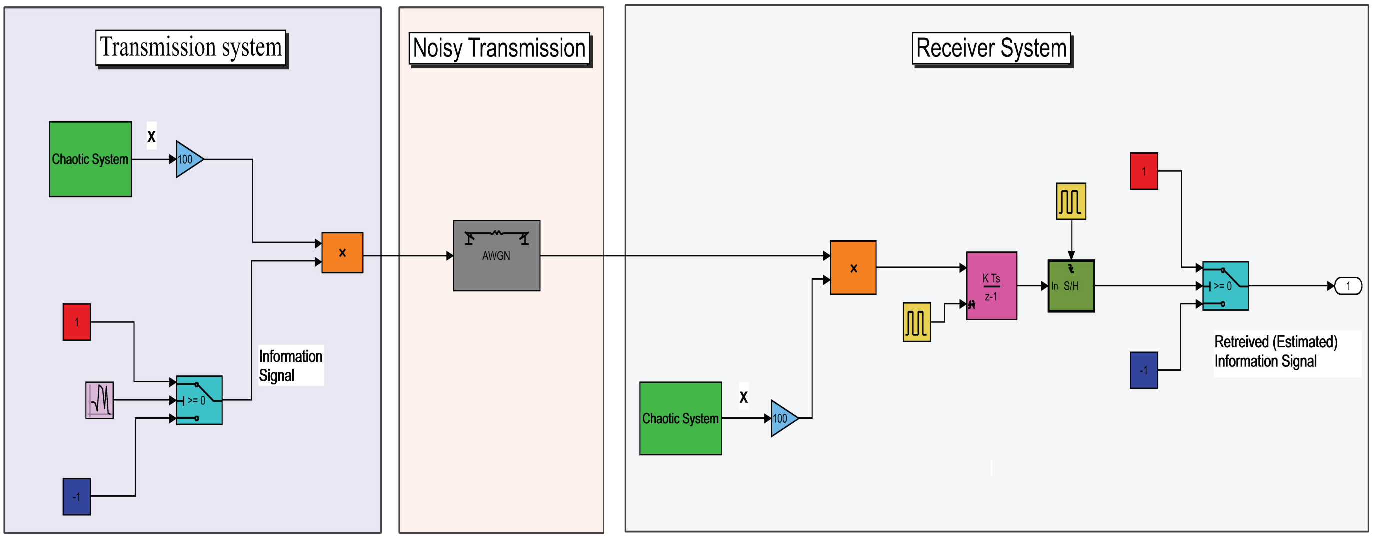


Fig. 6. SCSK secure communications design.

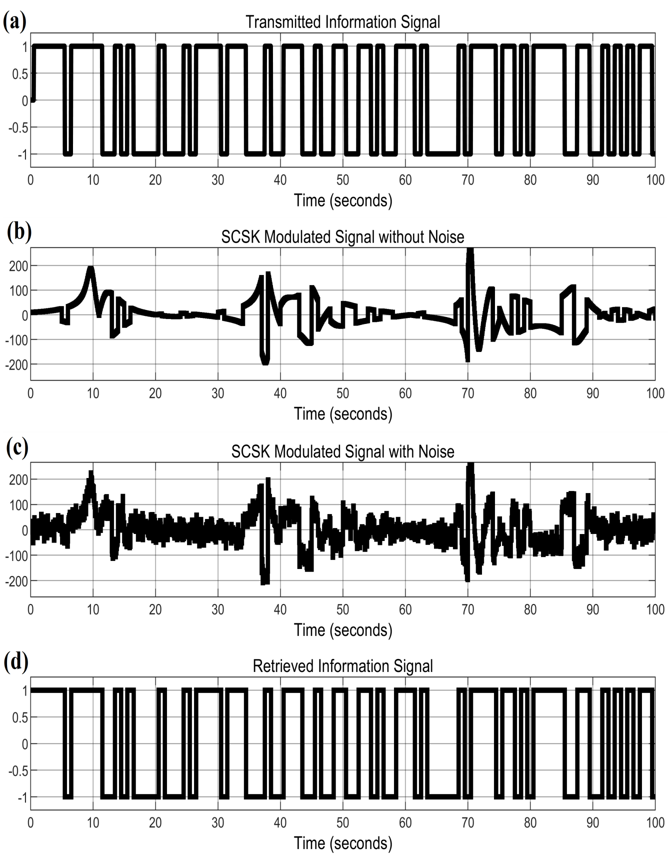


Fig. 7. Simulation results of the SCSK secure communications design. (a) The information signal to be transmitted securely, (b) the chaos modulated transmitted signal, (c) the noisy transmitted signal, (d) the information signal reconstructed at the receiver.


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